Diversity With Practical Channel Estimation

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***Abstract*—In this paper, we present a framework for evaluating the bit error probability of N_d-branch diversity combining in the presence of non-ideal channel estimates. The estimator structure presented is based on the maximum-likelihood (ML) estimate and arises naturally as the sample mean of N_p pilot symbols. The framework presented requires only the evaluation of a single integral involving the moment generating function of the norm square of the channel-gain vector, and is applicable to channels with arbitrary distribution, including correlated fading. Our analytical results show that the practical ML channel estimator preserves the diversity order of an N_d-branch diversity system, contrary to conclusions in the literature based upon a model that assumes a fixed correlation between the channel and its estimate. Finally, we investigate the asymptotic signal-to-noise ratio penalty due to estimation error and reveal a surprising lack of dependence on the number of diversity branches.**

***Index Terms*—Channel-state information, diversity, estimation error, imperfect channel knowledge, maximal-ratio combining, weighting errors.**

I. Introduction

DIVERSITY techniques can significantly improve the performance of wireless communication systems [1]–[4]. Among the various forms of diversity techniques, perfect coherent maximal-ratio combining (MRC) plays an important role as it provides the maximum instantaneous signal-to-noise ratio (SNR) at the combiner output. The performance of MRC over flat-fading channels has been extensively investigated in the literature. For example, multipath diversity using Rake reception with MRC has played an increasingly important role in spread-spectrum multiple-access systems [5]–[7] and more recently in third-generation wireless systems [8]–[10], as well as in ultrawide-bandwidth (UWB) systems [11]–[14]. These results assume perfect channel knowledge; however, practical receivers must estimate the channel, thereby incurring estimation error that needs to be accounted for in the performance analysis.

The problem of weighting error in what is essentially a maximal-ratio combiner was examined in [15], [16]. The system was assumed to be operating in independent identically distributed (i.i.d.) Rayleigh fading channels and estimates of the channel were derived from a pilot tone. The pilot tone was transmitted at a frequency offset from the data channels and used to provide appropriate weighting for combining. Expressions for the distribution of the instantaneous SNR,[1] as well as the error probability of both noncoherent and coherent binary orthogonal signaling schemes were developed in [15] and [16]. Similarly, [17] and [18] analyzed the distribution of the SNR in the presence of complex Gaussian weighting errors for MRC. In these studies, the weighting errors were characterized by a correlation coefficient between the channel gain and its estimate.

While the SNR is a meaningful measure for analog systems, it does not completely describe the performance of a digital system. A more meaningful measure for digital systems is the bit error probability (BEP). In [19]–[21], the BEP was derived for MRC systems by averaging the conditional BEP, conditioned on the SNR. Note however, that these results can be misleading as they do not truly reflect the actual BEP [15], [22]. The averaging in [19]–[21] was performed over a distribution developed from the SNR distribution given in [15]–[18]. The studies in [19]–[21] considered a model where the correlation coefficient between the channel estimate and the true channel is independent of the SNR, that is, the BEP was parameterized by fixed values of correlation.

Regardless of the choice of the model, one expects the accuracy of the estimator to improve as the SNR increases. Along these lines, [23]–[25] considered a different model for analyzing error probability in digital transmission systems using pilot signals in which the correlation coefficient between the channel estimate and the true channel is dependent on the SNR. This model reflects the fact that as the SNR increases, the estimator is capable of achieving a higher level of accuracy. Pilot symbol assisted modulation for single-antenna systems in time-varying Rayleigh fading channels has been analyzed assuming frequency-flat and frequency-selective channels in [26] and [27], respectively. Note that the work in [15]–[21], [23], and [24] was applicable only to i.i.d. Rayleigh fading environments.

In this paper, we develop an analytical framework that enables the evaluation of the performance of N_d-branch diversity systems with practical channel estimation. This framework is applicable to any environment, provided that its fading can be characterized by a moment generating function (MGF). Our methodology, requiring only the evaluation of a single integral with finite limits, is valid for channels with arbitrary

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[1]Throughout this paper, we use the term SNR to refer to instantaneous SNR. The term average SNR is explicitly used to describe the SNR averaged over the fading ensemble.

distribution, including correlated fading. To illustrate the proposed methodology, we consider Nakagami-m fading channels that have been shown to accurately model the amplitude distribution of the UWB indoor channel [28].[2] We also examine the case of Ricean fading, as it is appropriate for channels with line-of-sight components, such as satellite communication channels [3], [29]. We consider a channel estimator structure in which the correlation between the estimate and the true channel is a function that is dependent on the SNR. The SNR penalty, arising from degradation due to practical channel estimation, is quantified and we reveal a surprisingly small dependence on N_d.

This paper is organized as follows. In the next section, the models for both the system and estimator are presented. In Section III, we evaluate the BEP of N_d-branch diversity for channels with arbitrary fading distributions and discuss some special cases. We apply our BEP expressions to a few common channel models and develop asymptotic expressions in Section IV. In Section V, we discuss important aspects of practical diversity systems, including the correlation coefficient between the true channel gain and its estimate and the SNR penalty due to practical channel estimation. Finally, in Section VI, we present concluding remarks.

II. Model

We consider an N_d-branch diversity system utilizing a binary phase-shift keying (BPSK) signaling scheme, where in the interval $(0, T)$ we transmit signals of the form[3]

$$s_m(t) = \Re\left\{a_m g(t) \exp(j2\pi f_c t)\right\}, \qquad m = 0, 1 \tag{1}$$

where a_m denotes the data symbols taking the values ± 1. Here, the signal pulse shape $g(t)$ is a real-valued waveform that has energy $E_\mathrm{s} = (1/2)\int_0^T |g(t)|^2 \mathrm{d}t$ and support $(0, T)$. The received signal on the kth branch is then modeled as

$$r_k(t) = h_k s_m(t) + n_k(t), \qquad 0 \le t \le T, \quad 1 \le k \le N_\mathrm{d}. \tag{2}$$

The receiver demodulates $r_k(t)$ using the matched filter with impulse response $(1/\sqrt{2E_\mathrm{s}}) g^*(T - t)$.[4] Sampling the output yields

$$r_k = h_k s_m + n_k \tag{3}$$

where $s_m \in \{-\sqrt{2E_\mathrm{s}}, +\sqrt{2E_\mathrm{s}}\}$ represents the message symbol, h_k is a complex multiplicative gain introduced by fading in the channel, and n_k represents a sample of the additive noise on the kth branch. The additive noise is modeled as a complex Gaussian random variable (r.v.) with zero mean and variance N_0 per dimension and is assumed to be independent among the diversity branches. We consider slowly fading channels, so $\mathbf{h} = [\, h_1 \quad h_2 \quad \cdots \quad h_{N_\mathrm{d}} \,]$ is effectively constant over a block of symbols, without making any assumptions about the distribution of $\mathbf{h}$. Note also that there are no restrictions placed on the correlation between individual branch fading gains h_k, that is, our analysis is valid for channels with arbitrary correlation matrix.

If $\mathbf{h}$ were known at the receiver, the optimal combiner that maximizes the output SNR is well known to be MRC

$$r = \sum_{k=1}^{N_\mathrm{d}} h_k^* r_k.$$

In practice, however, $\mathbf{h}$ must be estimated; and thus the combiner output is

$$r = \sum_{k=1}^{N_\mathrm{d}} \hat{h}_k^* r_k \tag{4}$$

where $\hat{h}_k$ is an estimate of the multiplicative gain h_k on the kth branch. Clearly, the performance of this combining scheme greatly depends on the quality of the estimate $\hat{h}_k$.[5] As in [23]–[25], information can be derived from a pilot transmitted in previous signaling intervals to form an estimate of the channel. Without loss of generality, all pilot symbols are considered to be $+1$. The received pilot, after demodulation, matched filtering, and sampling can be represented by

$$p_{k,i} = \sqrt{2E_\mathrm{p}} h_k + n_{k,i} \tag{5}$$

where $p_{k,i}$ and $n_{k,i}$ denote the pilot symbol and noise samples, respectively, received on the kth branch during the ith previous signaling interval and E_p is the energy of the pilot symbol. Then, the linear estimate based on the previous N_p pilot transmissions is given by

$$\hat{h}_k = \frac{\sum_{i=1}^{N_\mathrm{p}} c_i p_{k,i}}{\sqrt{2E_\mathrm{p}} \sum_{i=1}^{N_\mathrm{p}} c_i} = h_k + \frac{\sum_{i=1}^{N_\mathrm{p}} c_i n_{k,i}}{\sqrt{2E_\mathrm{p}} \sum_{i=1}^{N_\mathrm{p}} c_i} \tag{6}$$

where c_i is an estimator weighting coefficient [30], [31]. The maximum-likelihood estimate arises if we let $c_i = 1$, $\forall i$, which gives[6]

$$\hat{h}_k = h_k + \frac{\sum_{i=1}^{N_\mathrm{p}} n_{k,i}}{\sqrt{2E_\mathrm{p}} N_\mathrm{p}}.$$

Note that this particular estimator structure is the sample average of N_p pilot transmissions. Furthermore, this estimator is both unbiased and efficient, with $\mathbb{E}\{\hat{h}_k\} = h_k$ and variance $\mathbb{E}\{|\hat{h}_k - h_k|^2\} = N_0/(E_\mathrm{p} N_\mathrm{p})$, achieving the Cramer–Rao lower bound with equality. It is also important to realize that both the pilot energy and the number of pilot symbols play a critical role in the performance of this estimator. As the pilot energy and/or the number of pilot symbols increase, the estimate becomes more accurate. That is, the estimate, and hence its correlation with the true channel gain, depends on both the average pilot SNR and the number of pilots N_p used to form the estimate [32]. Fig. 1 shows the diversity combining system utilizing practical channel estimation in detail.

[2] Note that the special case of $m = 1$ reduces to the classical Rayleigh fading channel.

[3] $\Re\{\cdot\}$ is used to denote the real part.

[4] The complex conjugate is denoted by $(\cdot)^*$.

[5] This receiver structure is similar to the one studied in [23]–[25] and is referred to as "fixed-reference coherent detection" in [15].

[6] In reality, knowledge of E_p is not needed since scaling $\hat{h}_k$ by any positive scalar does not affect the performance of the decision process in (7).

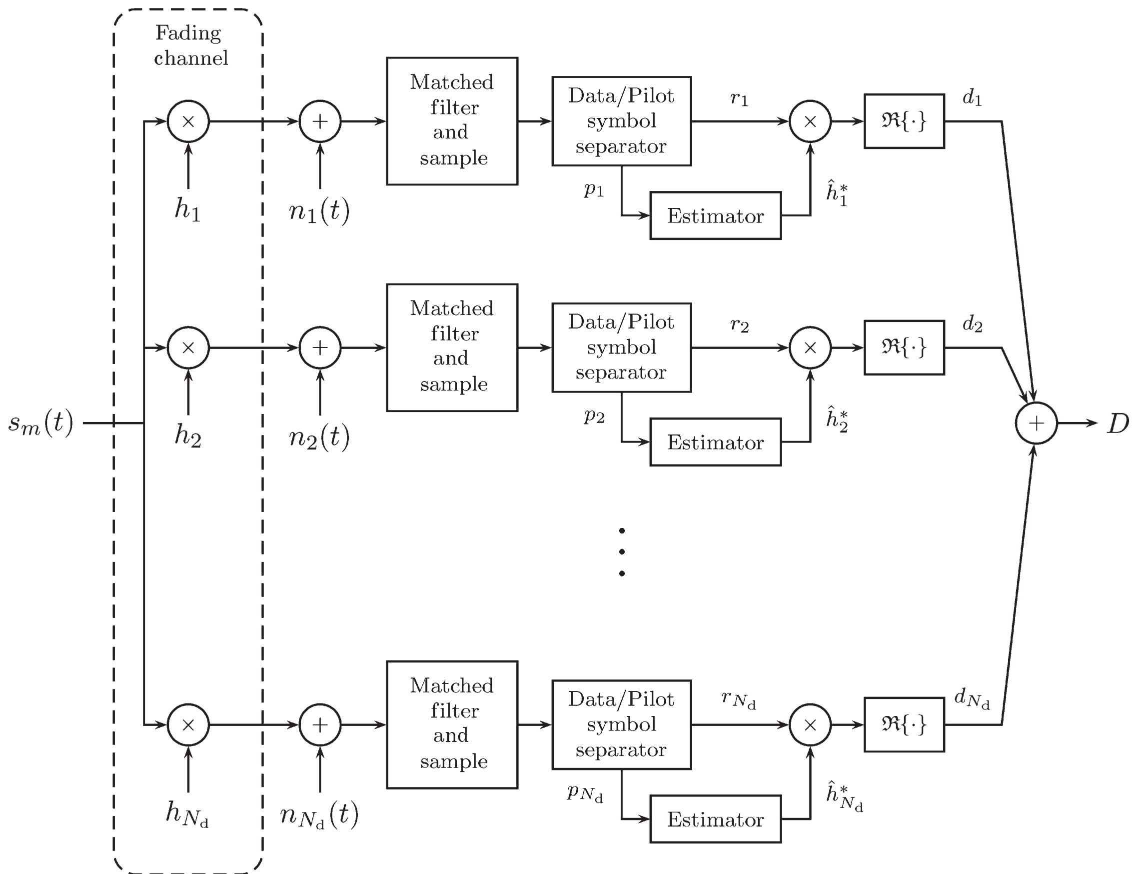

Fig. 1. Diversity system utilizing practical channel estimation.

III. Analysis

In this section, we determine the BEP via an MGF approach. We develop a methodology that requires evaluation of a single integral with finite limits and is applicable to channels with arbitrary distribution, including correlated fading.

A. Bit Error Probability Conditioned on $\mathbf{h}$

The decision variable is given by $D = \Re\{r\}$, which we can rewrite, using (4), as

$$D = \sum_{k=1}^{N_\mathrm{d}} d_k$$

where

$$\begin{aligned} d_k =& \Re\left\{\hat{h}_k^* r_k\right\} \\ =& \frac{1}{2}\left(h_k^* + e_k^*\right)\left(h_k s_m + n_k\right) \\ &+ \frac{1}{2}(h_k + e_k)\left(h_k^* s_m + n_k^*\right) \end{aligned} \quad (7)$$

and

$$e_k = \frac{1}{\sqrt{2E_\mathrm{p}} N_\mathrm{p}} \sum n_k$$

is the complex Gaussian estimation error. Given that $a_1 = +1$ was sent, an error will occur if $D < 0$. Thus, to evaluate the BEP, we need to determine $P_e = \Pr\{D < 0\}$.

In general, if the diversity branches are correlated, the variables d_k in (7) will not be independent. However, conditioned on the channel gain vector $\mathbf{h}$ the branches are conditionally independent and (7) can be viewed as a Hermitian quadratic form involving complex normal r.v.'s. [33].

The sum of r.v.'s, each given by the more general quadratic form

$$d_k = A|X_k|^2 + B|Y_k|^2 + CX_k^* Y_k + C^* X_k Y_k^*$$

was investigated in [23]–[25], [34], [35]. Examining (7), we observe that $A = B = 0$, $C = 1/2$, $X_k = h_k + e_k$, and $Y_k = h_k s_m + n_k$. Applying the result of [25, Appendix B], we obtain the conditional error probability, conditioned on the channel vector $\mathbf{h}$ as

$$\begin{aligned} \Pr\{e|\mathbf{h}\} =& Q_1(\zeta b, b) - \frac{1}{2} I_0(\zeta b^2) \\ &\times \exp\left(-\frac{b^2}{2}(1+\zeta^2)\right) + \frac{1}{2^{(2N_\mathrm{d}-1)}} \sum_{n=1}^{N_\mathrm{d}-1} I_n(\zeta b^2) \\ &\times \exp\left(-\frac{b^2}{2}(1+\zeta^2)\right) \sum_{k=0}^{N_\mathrm{d}-1-n} \binom{2N_\mathrm{d}-1}{k} [\zeta^{-n} - \zeta^n] \end{aligned} \quad (8)$$

where, as in [36], we define $\zeta \triangleq a/b$, $0 < \zeta \leq 1$, and

$$a = \frac{\sqrt{E_{\rm s}}\|\mathbf{h}\| \, |\sqrt{N_{\rm p}\varepsilon} - 1|}{\sqrt{2N_0}} \tag{9}$$

$$b = \frac{\sqrt{E_{\rm s}}\|\mathbf{h}\| \, (\sqrt{N_{\rm p}\varepsilon} + 1)}{\sqrt{2N_0}}. \tag{10}$$

Here, $\varepsilon \triangleq E_{\rm p}/E_{\rm s}$ is the ratio of pilot energy to data energy, and $\|\mathbf{h}\|^2 = \sum_{k=1}^{N_{\rm d}} |h_k|^2$ is the norm square of the channel-gain vector.

Now we make use of the following expressions for the Marcum Q-function, $Q_1(\zeta b, b)$, and the modified Bessel function of the nth order $I_n(z)$ [36], [37]

$$Q_1(\zeta b, b) = \frac{1}{4\pi}\int_{-\pi}^{\pi} \left\{ \exp\left(-\frac{b^2}{2}(1 + 2\zeta\sin\theta + \zeta^2)\right) + \exp\left(-\frac{b^2}{2}\left[\frac{(1-\zeta^2)^2}{1+2\zeta\sin\theta+\zeta^2}\right]\right)\right\} {\rm d}\theta \tag{11}$$

$$I_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos\left(n\left(\theta + \frac{\pi}{2}\right)\right) {\rm e}^{-z\sin\theta}{\rm d}\theta. \tag{12}$$

Application of (11) and (12) to (8) and further simplification yields

$$\Pr\{e|\mathbf{h}\} = \frac{1}{4\pi}\int_{-\pi}^{\pi} \left\{ \exp\left(-\frac{b^2}{2}\left[\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right]\right) + f(\theta;\zeta)\exp\left(-\frac{b^2}{2}g(\theta;\zeta)\right)\right\} {\rm d}\theta \tag{13}$$

where

$$f(\theta;\zeta) = \frac{1}{2^{(2N_{\rm d}-2)}}\sum_{n=1}^{N_{\rm d}-1}\cos\left(n\left(\theta+\frac{\pi}{2}\right)\right)[\zeta^{-n} - \zeta^n] \times \sum_{k=0}^{N_{\rm d}-1-n}\binom{2N_{\rm d}-1}{k}$$

$$g(\theta;\zeta) = 1 + 2\zeta\sin\theta + \zeta^2.$$

Now, we note that

$$b^2 = \frac{E_{\rm s}\|\mathbf{h}\|^2(\sqrt{N_{\rm p}\varepsilon}+1)^2}{2N_0} = \frac{\Gamma_{\rm tot}\|\mathbf{h}\|^2(\sqrt{N_{\rm p}\varepsilon}+1)^2}{2\mathbb{E}\{\|\mathbf{h}\|^2\}} \tag{14}$$

$$\zeta = \frac{|\sqrt{N_{\rm p}\varepsilon} - 1|}{\sqrt{N_{\rm p}\varepsilon} + 1} \tag{15}$$

where we have defined $\Gamma_{\rm tot} \triangleq \mathbb{E}\{\|\mathbf{h}\|^2\}(E_{\rm s}/N_0)$ as the average total SNR. Substitution of (14) and (15) into (13) yields the simplified expression for the BEP when conditioned on the channel shown in (16) at the bottom of the page. The advantage of (16), compared to the original equation (8), is now apparent in that averaging over $\mathbf{h}$ is a simple process because it lies only in the exponents. In addition, (16) only depends on $\|\mathbf{h}\|^2$, that is, it is sufficient to only condition on a single r.v., namely the norm square of the channel gain vector, as opposed to conditioning on the entire channel vector, involving $N_{\rm d}$ r.v.'s.

B. Bit Error Probability for Arbitrary Fading Channels

We now determine the BEP of our practical diversity system in arbitrary fading channels by averaging (16) over the channel ensemble

$$P_e = \mathbb{E}_{\mathbf{h}}\{\Pr\{e|\mathbf{h}\}\}.$$

In [19]–[21], the conditional BEP, conditioned on the SNR, is averaged over the distribution of the SNR. Our derivation shows that one must average $\Pr\{e|\mathbf{h}\}$ in (16) over the distribution of the fading ensemble to get the exact BEP. This is in agreement with the observation made recently in [22]. A similar observation was also made more than four decades ago in [15]: "Since we do not have exact coherent detection one cannot average over the nonfading coherent detection error probability ... to obtain the error probability of fixed-reference coherent detection."

Since the $N_{\rm d}$ terms that we are averaging over appear only as $\|\mathbf{h}\|^2$ in the exponents of $\Pr\{e|\mathbf{h}\}$ in (16), we obtain the exact BEP expression as shown in (17) at the bottom of the page,

$$\Pr\{e|\mathbf{h}\} = \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{\exp\left(-\frac{\Gamma_{\rm tot}\|\mathbf{h}\|^2(\sqrt{N_{\rm p}\varepsilon}+1)^2}{4\mathbb{E}\{\|\mathbf{h}\|^2\}}\left[\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right]\right) + f(\theta;\zeta)\exp\left(-\frac{\Gamma_{\rm tot}\|\mathbf{h}\|^2(\sqrt{N_{\rm p}\varepsilon}+1)^2}{4\mathbb{E}\{\|\mathbf{h}\|^2\}}g(\theta;\zeta)\right)\right\}{\rm d}\theta \tag{16}$$

$$P_e(\Gamma_{\rm tot}) = \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{M_{\|\mathbf{h}\|^2}\left(-\frac{\Gamma_{\rm tot}(\sqrt{N_{\rm p}\varepsilon}+1)^2}{4\mathbb{E}\{\|\mathbf{h}\|^2\}}\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right) + f(\theta;\zeta)M_{\|\mathbf{h}\|^2}\left(-\frac{\Gamma_{\rm tot}(\sqrt{N_{\rm p}\varepsilon}+1)^2}{4\mathbb{E}\{\|\mathbf{h}\|^2\}}g(\theta;\zeta)\right)\right\}{\rm d}\theta \tag{17}$$

where $M_{\|\mathbf{h}\|^2}(s) \triangleq \mathbb{E}_{\mathbf{h}}\{e^{s\|\mathbf{h}\|^2}\}$. Thus, we have an exact BEP expression for practical diversity systems in the presence of channel estimation error, for arbitrary channels. All we require is the evaluation of a single integral with finite limits and an integrand involving only the MGF of the norm square of the channel-gain vector.

C. Special Cases

In this section, we consider some special cases of the BEP expression. From (17), we see that the BEP depends on N_p and ε through the quantity $N_\mathrm{p}\varepsilon$. Here, we investigate the cases where $N_\mathrm{p}\varepsilon$ is large, $N_\mathrm{p}\varepsilon \to 1$, and when $N_\mathrm{p}\varepsilon = 0$.

1) Large $N_\mathrm{p}\varepsilon$: Since N_p and ε represent the number of pilot symbols used to form the estimate of the channel and the ratio of pilot energy to data energy, respectively, with increasing N_p and/or ε, we expect to see performance approach that of perfect channel knowledge. From (15), as $N_\mathrm{p}\varepsilon \to \infty$, we note that $\zeta \to 1$. This causes $f(\theta;\zeta)$ to go to zero, hence the second term in (17) does not contribute to the integral. After some simplification, we have

$$P_e(\Gamma_{\mathrm{tot}}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_{\|\mathbf{h}\|^2}\left(-\frac{\Gamma_{\mathrm{tot}}}{\mathbb{E}\{\|\mathbf{h}\|^2\}\sin^2\theta}\right)d\theta \qquad (18)$$

as $N_\mathrm{p}\varepsilon \to \infty$. We recognize (18) as the BEP for BPSK with perfect channel knowledge [36, p. 268].

2) $N_\mathrm{p}\varepsilon \to 1$: This case is of interest as it includes the simplest estimator, namely the case where $N_\mathrm{p} = 1$ and $\varepsilon \to 1$. From (15), when $N_\mathrm{p}\varepsilon \to 1$, $\zeta \to 0$. In order to evaluate the BEP performance in this case, we begin with (8) and apply the small argument form of the modified Bessel function of the nth order [36, p. 84]

$$I_n(z) \approx \frac{\left(\frac{z}{2}\right)^n}{n!}, \quad z \text{ small}$$

where we have assumed that n is a nonnegative integer. Noting that $Q_1(0,b) = \exp(-b^2/2)$, after careful simplification we have

$$\Pr\{e|\mathbf{h}\} = \frac{1}{2^{2N_\mathrm{d}-1}}\sum_{n=0}^{N_\mathrm{d}-1}\frac{1}{n!}\left(\frac{b^2}{2}\right)^n\exp\left(-\frac{b^2}{2}\right) \times \sum_{k=0}^{N_\mathrm{d}-1-n}\binom{2N_\mathrm{d}-1}{k}.$$

Applying properties of MGFs, we obtain the unconditional BEP expression as

$$P_e(\Gamma_{\mathrm{tot}}) = \frac{1}{2^{2N_\mathrm{d}-1}}\sum_{n=0}^{N_\mathrm{d}-1}\frac{1}{n!}\left(\frac{\Gamma_{\mathrm{tot}}}{\mathbb{E}\{\|\mathbf{h}\|^2\}}\right)^n \times \frac{d^n}{ds^n}M_{\|\mathbf{h}\|^2}(s)\Big|_{s=-\frac{\Gamma_{\mathrm{tot}}}{\mathbb{E}\{\|\mathbf{h}\|^2\}}}\sum_{k=0}^{N_\mathrm{d}-1-n}\binom{2N_\mathrm{d}-1}{k}.$$

3) $N_\mathrm{p}\varepsilon = 0$: In this case, no channel estimation is performed, so we expect performance to degrade completely. From (15), when $N_\mathrm{p}\varepsilon = 0$ we have $\zeta = 1$. This causes $f(\theta;\zeta)$ to always equal zero, hence the second term in (17) does not contribute to the integral. Note also that the argument of the MGF in the first term of (17) is also zero. Using the fact that $M_{\|\mathbf{h}\|^2}(0) = 1$, we have $P_e(\Gamma_{\mathrm{tot}}) = (1/4\pi)\int_{-\pi}^{\pi} d\theta = 1/2$. As expected, without performing any estimation the receiver achieves the worst possible performance.

IV. BEP FOR SPECIFIC FADING DISTRIBUTIONS

Using the analytical framework developed in the previous section, which is valid for arbitrary fading distributions, we evaluate the BEP for some common channel models. For illustrative purposes we consider only independent nonidentically distributed channels, even though our framework is applicable to correlated channels. First, we consider Nakagami-m distributed channels with arbitrary m parameters. Then, the case of Rayleigh fading is presented. The case of Ricean distributed channels is also considered. We obtain asymptotic results for the special case of i.i.d. fading to determine the diversity order of diversity systems with practical channel estimation in these channels.

A. Nakagami and Rayleigh Fading Environments

Nakagami-m fading channels have received considerable attention in the study of various aspects of wireless systems [38]–[41]. In particular, it was shown recently that the amplitude distribution of the resolved multipaths in UWB indoor channels can be well modeled by the Nakagami-m distribution [28]. The Nakagami-m family of distributions, also known as the "m-distribution," contains Rayleigh fading $(m = 1)$ as a special case; along with cases of fading that are more severe than Rayleigh $(1/2 \leq m < 1)$ as well as cases less severe than Rayleigh $(m > 1)$.

In a Nakagami fading environment, each $|h_k|$ has a Nakagami distribution with parameter $m_k \geq 1/2$. The MGF for the sum of the squares of such r.v.'s is given by

$$M_{\|\mathbf{h}\|^2}(s) = \prod_{k=1}^{N_\mathrm{d}}\left[\frac{1}{1 - s\frac{\mathbb{E}\{|h_k|^2\}}{m_k}}\right]^{m_k}. \qquad (19)$$

Rayleigh fading can be obtained by setting $m_k = 1$, $\forall k$ in the Nakagami-m model above.

B. Ricean Fading Environment

The Rice distribution is appropriate for modeling communication environments where there are line-of-sight components, such as satellite channels [3], [29]. Using a procedure similar to [42], we can derive the MGF of the norm square of the channel-gain vector in a Ricean fading environment. In such an environment, each h_k has a complex Gaussian distribution with nonzero mean. The MGF of the norm square of the channel gains $\|\mathbf{h}\|^2$ is given by

$$M_{\|\mathbf{h}\|^2}(s) = \prod_{k=1}^{N_\mathrm{d}} \left[\frac{\kappa_k}{\kappa_k - s|\mu_k|^2}\right] \exp\left(\frac{s\kappa_k|\mu_k|^2}{\kappa_k - s|\mu_k|^2}\right) \quad (20)$$

where each $\mu_k = \mathbb{E}\{h_k\}$, and $\kappa_k \triangleq |\mu_k|^2/(\mathbb{E}\{|h_k|^2\} - |\mu_k|^2)$ is the Rice factor. Note that μ_k is complex in general, but the distribution of $\|\mathbf{h}\|^2$ does not depend on the phase of each μ_k.

C. Asymptotic Results

1) Nakagami Fading: We now consider the behavior of our expressions in (17) and (18) as the SNR increases asymptotically for the case of i.i.d. Nakagami fading channels with $\mathbb{E}\{|h_k|^2\} = 1$. In this case, the MGF becomes

$$M_{\|\mathbf{h}\|^2}(s) = \left(\frac{1}{1-\frac{s}{m}}\right)^{mN_\mathrm{d}} \approx \left(-\frac{m}{s}\right)^{mN_\mathrm{d}} \quad (21)$$

where the approximation is for large s. Using (21) in (17), one can obtain the asymptotic behavior for the case of imperfect channel knowledge as $\Gamma_\mathrm{tot} \to \infty$

$$\begin{aligned} P_{e,{\text{Asym-I} \atop \text{Nakagami}}}(\Gamma_\mathrm{tot}) &= \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{\left[\frac{m\mathbb{E}\{\|\mathbf{h}\|^2\}g(\theta;\zeta)}{\frac{\Gamma_\mathrm{tot}}{4}(\sqrt{N_\mathrm{p}\varepsilon}+1)^2(1-\zeta^2)^2}\right]^{mN_\mathrm{d}} \right. \\ &\quad \left. + f(\theta;\zeta)\left[\frac{m\mathbb{E}\{\|\mathbf{h}\|^2\}}{\frac{\Gamma_\mathrm{tot}}{4}(\sqrt{N_\mathrm{p}\varepsilon}+1)^2 g(\theta;\zeta)}\right]^{mN_\mathrm{d}}\right\}\mathrm{d}\theta \\ &= K_\mathrm{I,Nakagami}(m, N_\mathrm{d}, N_\mathrm{p}, \varepsilon)\left(\frac{1}{\Gamma_\mathrm{tot}}\right)^{mN_\mathrm{d}} \end{aligned} \quad (22)$$

where we have defined

$$\begin{aligned} K_\mathrm{I,Nakagami}(m, N_\mathrm{d}, N_\mathrm{p}, \varepsilon) &\triangleq \frac{1}{4\pi}\left[\frac{4mN_\mathrm{d}}{(\sqrt{N_\mathrm{p},\varepsilon}+1)^2}\right]^{mN_\mathrm{d}} \\ &\quad \times \int_{-\pi}^{\pi}\left\{\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{mN_\mathrm{d}} + \frac{f(\theta;\zeta)}{[g(\theta;\zeta)]^{mN_\mathrm{d}}}\right\}\mathrm{d}\theta. \end{aligned} \quad (23)$$

In (23), we have used the fact that $\mathbb{E}\{\|\mathbf{h}\|^2\} = N_\mathrm{d}\mathbb{E}\{|h|^2\} = N_\mathrm{d}$. The subscript Asym-I is used to denote the asymptotic behavior with imperfect channel knowledge.

Using (21) in (18), one can similarly derive the asymptotic behavior for the case of perfect channel knowledge as

$$P_{e,{\text{Asym-P} \atop \text{Nakagami}}}(\Gamma_\mathrm{tot}) = K_\mathrm{P,Nakagami}(m, N_\mathrm{d})\left(\frac{1}{\Gamma_\mathrm{tot}}\right)^{mN_\mathrm{d}} \quad (24)$$

where

$$K_\mathrm{P,Nakagami}(m, N_\mathrm{d}) \triangleq \frac{(mN_\mathrm{d})^{mN_\mathrm{d}}\Gamma\left(\frac{1}{2}+mN_\mathrm{d}\right)}{2\sqrt{\pi}\Gamma(1+mN_\mathrm{d})} \quad (25)$$

and $\Gamma(\cdot)$ is the gamma function[7]

$$\Gamma(p) \triangleq \int_0^\infty t^{p-1}\mathrm{e}^{-t}\mathrm{d}t.$$

The subscript Asym-P is used to denote the asymptotic behavior with perfect channel knowledge.

2) Rayleigh Fading: For the special case of Rayleigh fading, the asymptotic results can be derived by setting $m = 1$ in (22) and (24). In doing this we have

$$P_{e,{\text{Asym-I} \atop \text{Rayleigh}}}(\Gamma_\mathrm{tot}) = K_\mathrm{I,Rayleigh}(N_\mathrm{d}, N_\mathrm{p}, \varepsilon)\left(\frac{1}{\Gamma_\mathrm{tot}}\right)^{N_\mathrm{d}} \quad (26)$$

where $K_\mathrm{I,Rayleigh}(N_\mathrm{d}, N_\mathrm{p}, \varepsilon) \triangleq K_\mathrm{I,Nakagami}(1, N_\mathrm{d}, N_\mathrm{p}, \varepsilon)$. Similarly

$$P_{e,{\text{Asym-P} \atop \text{Rayleigh}}}(\Gamma_\mathrm{tot}) = K_\mathrm{P,Rayleigh}(N_\mathrm{d})\left(\frac{1}{\Gamma_\mathrm{tot}}\right)^{N_\mathrm{d}} \quad (27)$$

where $K_\mathrm{P,Rayleigh}(N_\mathrm{d}) \triangleq K_\mathrm{P,Nakagami}(1, N_\mathrm{d})$.

3) Ricean Fading: Similar to the case above, we consider the asymptotic behavior of (17) and (18) in i.i.d. Ricean fading with $\mathbb{E}\{|h|^2\} = 1$. In this case the MGF becomes

$$\begin{aligned} M_{\|\mathbf{h}\|^2}(s) &= \left[\frac{1+\kappa}{1+\kappa-s}\right]^{N_\mathrm{d}} \exp\left(\frac{sN_\mathrm{d}\kappa}{1+\kappa-s}\right) && (28) \\ &\approx \left[-\frac{1+\kappa}{s}\right]^{N_\mathrm{d}} \exp(-N_\mathrm{d}\kappa) && (29) \end{aligned}$$

where κ is the Rice factor and the approximation is valid for large values of s. Using (29) in (17), one can obtain the asymptotic behavior for the case of imperfect channel knowledge in Ricean fading as $\Gamma_\mathrm{tot} \to \infty$

$$\begin{aligned} P_{e,{\text{Asym-I} \atop \text{Ricean}}}(\Gamma_\mathrm{tot}) &= \left[\frac{N_\mathrm{d}(1+\kappa)\mathrm{e}^{-\kappa}}{\Gamma_\mathrm{tot}}\right]^{N_\mathrm{d}} \frac{1}{4\pi}\left[\frac{4}{(\sqrt{N_\mathrm{p}\varepsilon}+1)^2}\right]^{N_\mathrm{d}} \\ &\quad \times \int_{-\pi}^{\pi}\left\{\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{N_\mathrm{d}} + \frac{f(\theta;\zeta)}{[g(\theta;\zeta)]^{N_\mathrm{d}}}\right\}\mathrm{d}\theta \\ &= K_\mathrm{I,Ricean}(\kappa, N_\mathrm{d}, N_\mathrm{p}, \varepsilon)\left(\frac{1}{\Gamma_\mathrm{tot}}\right)^{N_\mathrm{d}} \end{aligned} \quad (30)$$

[7]We use the $\Gamma(\cdot)$ to denote the gamma function, while Γ_tot denotes the average total SNR.

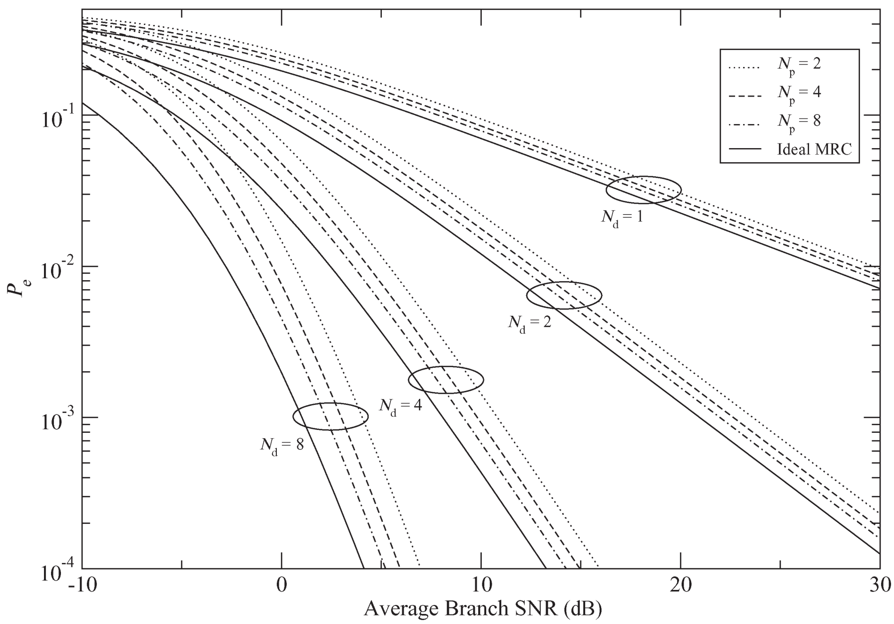

Fig. 2. Performance of BPSK in i.i.d. Nakagami fading with $m = 0.5$, for various $N_{\rm d}$, $N_{\rm p}$.

where we have defined

$$K_{\rm I,Ricean}(\kappa, N_{\rm d}, N_{\rm p}, \varepsilon) \triangleq \frac{1}{4\pi}\left[\frac{4N_{\rm d}(1+\kappa){\rm e}^{-\kappa}}{(\sqrt{N_{\rm p}\varepsilon}+1)^2}\right]^{N_{\rm d}} \times \int_{-\pi}^{\pi}\left\{\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{N_{\rm d}} + \frac{f(\theta;\zeta)}{[g(\theta;\zeta)]^{N_{\rm d}}}\right\}{\rm d}\theta. \quad (31)$$

A similar calculation using (29) in (18) yields the asymptotic behavior for perfect channel knowledge in Ricean fading

$$P_{e_{\rm Asym\text{-}P,\,Ricean}}(\Gamma_{\rm tot}) = \left[\frac{N_{\rm d}(1+\kappa){\rm e}^{-\kappa}}{\Gamma_{\rm tot}}\right]^{N_{\rm d}} \frac{\Gamma\left(\frac{1}{2}+N_{\rm d}\right)}{2\sqrt{\pi}\Gamma(1+N_{\rm d})} = K_{\rm P,Ricean}(\kappa, N_{\rm d})\left(\frac{1}{\Gamma_{\rm tot}}\right)^{N_{\rm d}} \quad (32)$$

where

$$K_{\rm P,Ricean}(\kappa, N_{\rm d}) \triangleq \frac{[N_{\rm d}(1+\kappa){\rm e}^{-\kappa}]^{N_{\rm d}}\,\Gamma\left(\frac{1}{2}+N_{\rm d}\right)}{2\sqrt{\pi}\Gamma(1+N_{\rm d})}. \quad (33)$$

Note that the results in (30) and (32) differ from their counterparts in Rayleigh fading, (26) and (27), by only the multiplicative factor $[(1+\kappa){\rm e}^{-\kappa}]^{N_{\rm d}}$.

For the case of Nakagami fading, it is clear from (22) and (24) that regardless of the number of pilot symbols used in the formation of an estimate of the channel, a diversity order of $mN_{\rm d}$ is still maintained as in the case of ideal MRC. Similarly, for the case of Ricean fading, (30) and (32) show that a diversity order of $N_{\rm d}$ is preserved. This behavior, arising purely from our analytical asymptotic expressions, is also evident from our numerical results as we will show in the next section. These results are in contrast to the analytical results presented in [19]–[21] which showed that, even with the estimate arbitrarily close to the ideal one, the asymptotic BEP is proportional to $1/\Gamma_{\rm tot}$. That is, even with an arbitrarily good estimate, diversity order is that of a single-branch system. For example, the expression [19, eq. (20)] shows that the diversity order is equal to that of a single-branch system.

V. Discussion and Numerical Results

In this section, we discuss aspects of the correlation coefficient between the true channel gain and its estimate, including the relation of this correlation coefficient to the SNR and the number of pilot symbols. We also examine the SNR penalty due to channel estimation error and give some numerical results.

A. Relationship Between Estimate Correlation, SNR, and $N_{\rm p}$

The correlation coefficient of the channel gain estimate with the true channel gain plays a crucial role in the performance of diversity systems with practical channel estimation. Here, we have used an estimator structure that employs pilot symbol transmission. The correlation coefficient that arises from such an estimator is given by

$$\rho_k = \frac{\mathbb{E}\left\{h_k\hat{h}_k^*\right\} - \mathbb{E}\{h_k\}\mathbb{E}\left\{\hat{h}_k^*\right\}}{\sqrt{\mathbb{E}\left\{|h_k-\mathbb{E}\{h_k\}|^2\right\}\mathbb{E}\left\{\left|\hat{h}_k-\mathbb{E}\{\hat{h}_k\}\right|^2\right\}}} = \begin{cases} \frac{\sqrt{N_{\rm p}\varepsilon}}{\sqrt{N_{\rm p}\varepsilon+\frac{1}{\Gamma_k}}}, & \text{Nakagami-}m\text{ fading} \\ \frac{\sqrt{N_{\rm p}\varepsilon}}{\sqrt{N_{\rm p}\varepsilon+\frac{1+\kappa_k}{\Gamma_k}}}, & \text{Ricean fading} \end{cases}$$

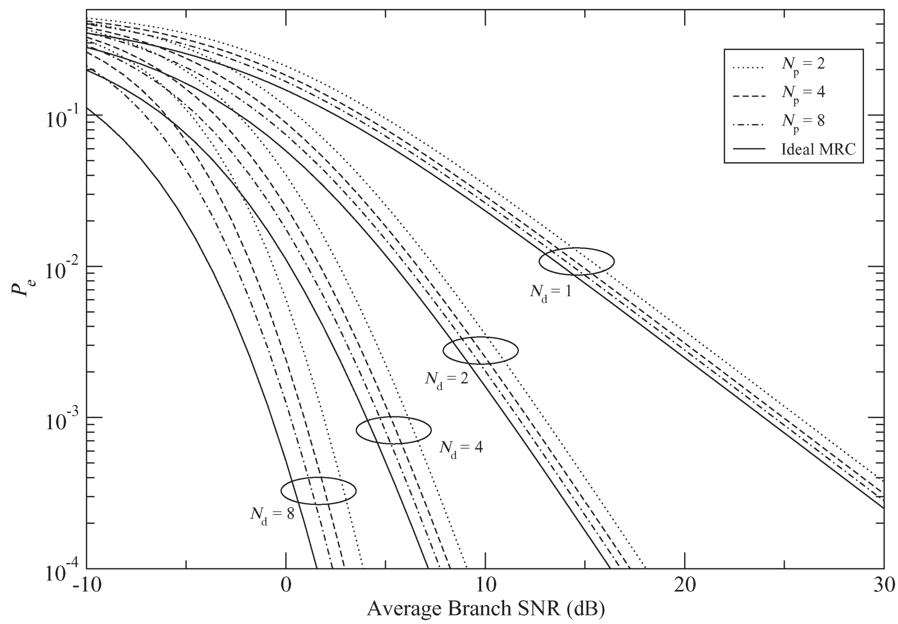

Fig. 3. Performance of BPSK in i.i.d. Nakagami fading with $m = 1$ (Rayleigh fading), for various N_d, N_p.

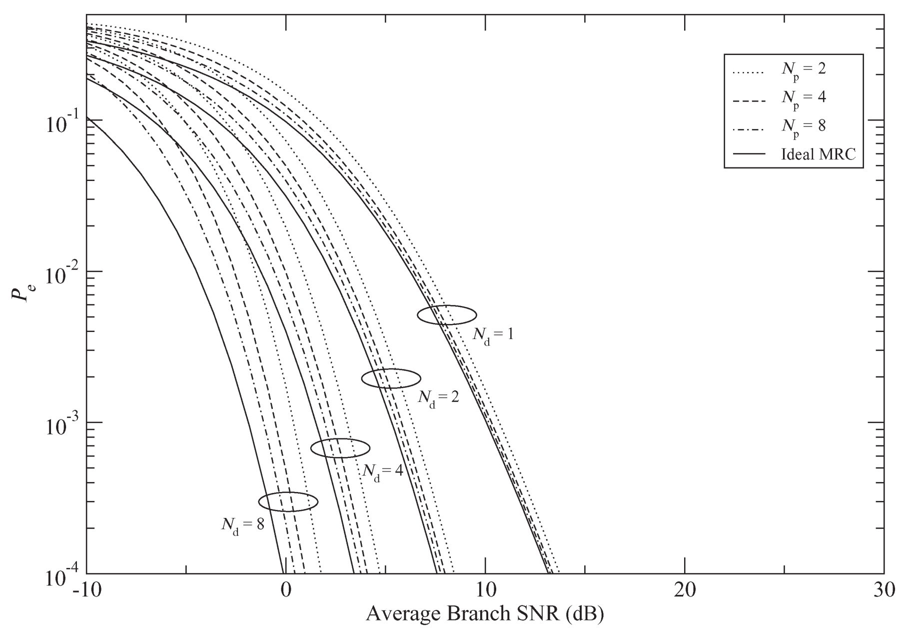

Fig. 4. Performance of BPSK in i.i.d. Nakagami fading with $m = 4$, for various N_d, N_p.

where $\Gamma_k \triangleq \mathbb{E}\{|h_k|^2\}(E_\mathrm{s}/N_0)$ is the average SNR on the kth diversity branch. It is important to note here that ρ_k is a function of the average branch SNR Γ_k as well as the number of pilot symbols N_p. As Γ_k tends toward the high SNR regime, the correlation approaches one. This fact makes intuitive sense, if a system is operating under high SNR, it should be able to achieve better accuracy in its estimate. This model is significantly different from other correlation models [17]–[21], where the correlation coefficient is explicitly set to a particular value, irrespective of the branch SNR. Similarly, as the number of pilot symbols used to form the estimate increases, the correlation approaches one. Naturally, as the number of channel measurements increases, we expect our knowledge of the channel to become more accurate.

Figs. 2–4 show the BEP for Nakagami fading for the cases where $m = 0.5$, $m = 1$, and $m = 4$, respectively, and $\varepsilon = 1$. In each case, note that the diversity order is preserved, regardless of the number of pilot symbols used in the estimation process. Also, note that as N_p increases, performance approaches that of perfect channel knowledge. Similar results are shown in Figs. 5 and 6 for a Ricean fading environment, with $\kappa = 5$ dB and $\kappa = 10$ dB, respectively, and $\varepsilon = 1$. These results are in agreement with our asymptotic analytical results in (22), (26), and (30), respectively. Previous numerical results

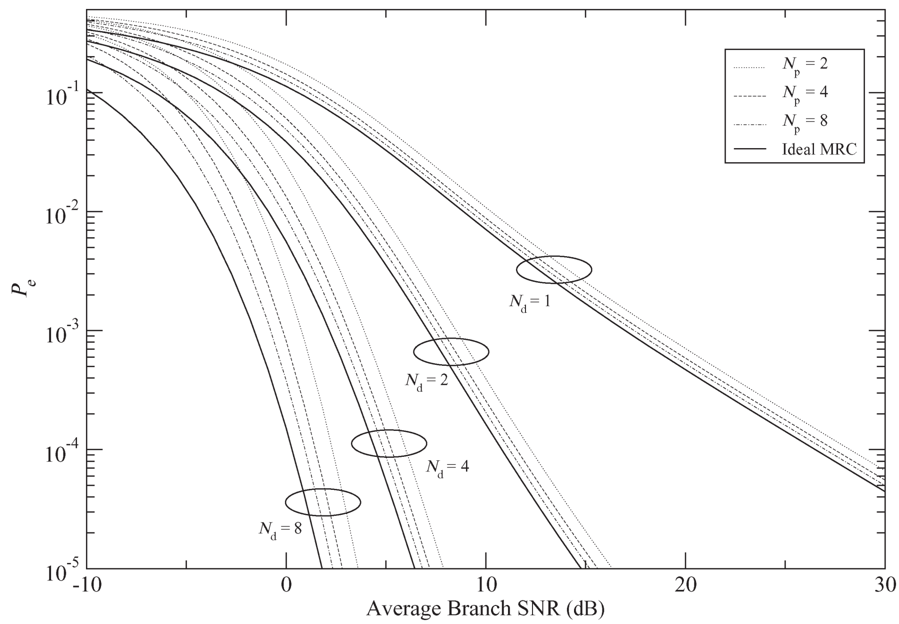

Fig. 5. Performance of BPSK in i.i.d. Ricean fading with $\kappa = 5$ dB, for various $N_{\rm d}$, $N_{\rm p}$.

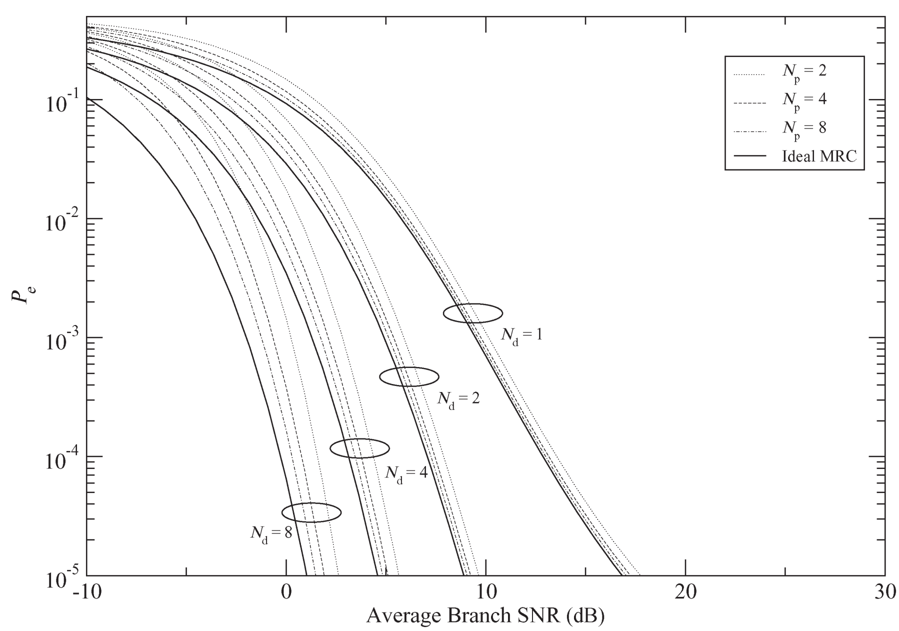

Fig. 6. Performance of BPSK in i.i.d. Ricean fading with $\kappa = 10$ dB, for various $N_{\rm d}$, $N_{\rm p}$.

in [15], [16], and [19]–[21] were only valid for i.i.d. Rayleigh fading environments, and showed that the diversity order was not preserved. For example, in [19], numerical results with $\rho = 0.9, 0.99, 0.999$ ($\rho = 1$ corresponds to an ideal estimate) all display asymptotic behavior of a single-branch system. Similar behavior can also be found in [21, Fig. 6].

Choosing the number of pilot symbols to use in the channel estimation is an important aspect of system design. Clearly, the number of pilot symbols cannot be arbitrarily large. The choice is governed foremost by the coherence time of the channel, and then by the requirements of the communication system in terms of bit rates and transmission power. Throughout, we have considered slowly fading channels in which a block of symbols experiences the same fading condition. Provided that the data symbols and the corresponding pilot symbols used to form an estimate are within the coherence time, the performance should follow what we have given above.

B. SNR Penalty

In comparison to ideal MRC, diversity with practical channel estimation will incur a loss in SNR, due to the fact that completely coherent combining is not possible. For analog systems, the SNR penalty is defined in terms of the degradation

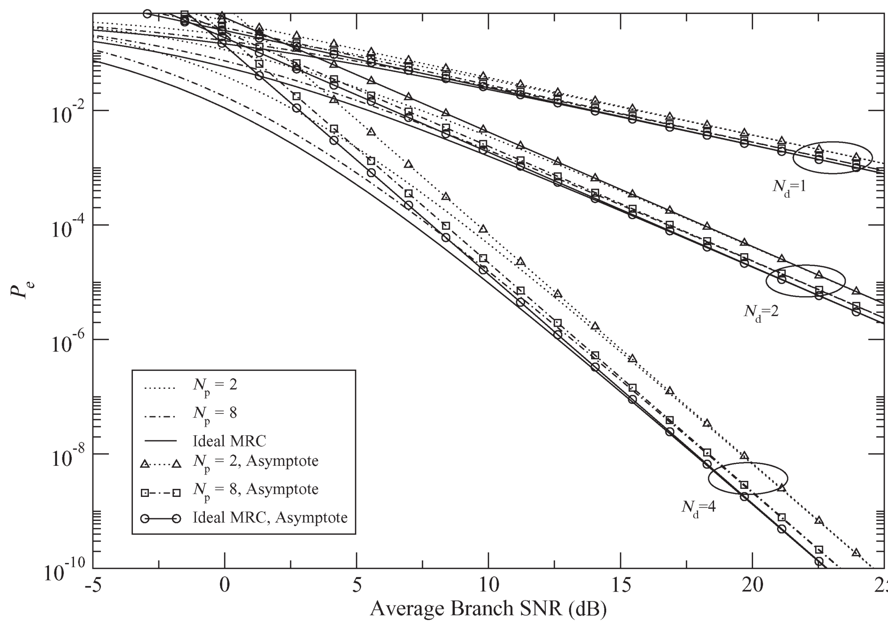

Fig. 7. Comparison of exact performance with asymptotic performance of BPSK in i.i.d. Nakagami fading with $m = 1$ (Rayleigh fading), for various N_d, N_p.

in the SNR. Instead, as in [43], we consider a measure that is more suitable for digital systems; the SNR penalty required to maintain a target BEP.

For a digital communication system, we define the SNR penalty β as the increase in SNR required for a diversity system with practical channel estimation to achieve the same target BEP as ideal MRC. Implicitly, we have

$$P_{e,\mathrm{I}}(\beta\Gamma_\mathrm{tot}) = P_{e,\mathrm{MRC}}(\Gamma_\mathrm{tot})$$

where $P_{e,\mathrm{I}}(\cdot)$, $P_{e,\mathrm{MRC}}(\cdot)$, β, and Γ_tot are the BEP for diversity combining with imperfect channel knowledge, the BEP for ideal MRC, the SNR penalty, and the total average SNR, respectively.

Note that the SNR penalty is a function of the target BEP, and therefore a function of the average SNR; that is, $\beta = \beta(\Gamma_\mathrm{tot})$. A closed-form expression for β is difficult to obtain, if at all possible. However, using (22) and (24), we can derive the asymptotic SNR penalty β_A for large SNR, such that

$$P_{e,\mathrm{Asym\text{-}I}}(\beta_\mathrm{A}\Gamma_\mathrm{tot}) = P_{e,\mathrm{Asym\text{-}P}}(\Gamma_\mathrm{tot}).$$

Solving this relation for the specific case of Nakagami fading channels gives

$$\beta_\mathrm{A} = \left[\frac{K_{\mathrm{I,Nakagami}}(m, N_\mathrm{d}, N_\mathrm{p}, \varepsilon)}{K_{\mathrm{P,Nakagami}}(m, N_\mathrm{d})}\right]^{\frac{1}{mN_\mathrm{d}}}$$
$$= \frac{4}{(\sqrt{N_\mathrm{p}\varepsilon}+1)^2}\left\{\frac{\Gamma(1+mN_\mathrm{d})}{2\sqrt{\pi}\Gamma\left(\frac{1}{2}+mN_\mathrm{d}\right)}\right.$$
$$\left.\times\int_{-\pi}^{\pi}\left\{\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{mN_\mathrm{d}} + \frac{f(\theta;\zeta)}{[g(\theta;\zeta)]^{mN_\mathrm{d}}}\right\}\mathrm{d}\theta\right\}^{\frac{1}{mN_\mathrm{d}}}. \quad (34)$$

Clearly, the asymptotic SNR penalty for Rayleigh fading is given by (34) when $m = 1$. A similar expression for Ricean fading can be derived using (30)–(33). Since (31) and (33) only differ by a multiplicative constant from (23) and (25) when $m = 1$, the asymptotic SNR penalty in Ricean fading is given by (34) with $m = 1$.

Figs. 7 and 8 show[8] the asymptotic BEP given by (22) and (30). These figures provide further confirmation that the practical channel-estimation scheme preserves the diversity order. From these figures, we see that the performance given by the asymptotic expressions quickly approaches the exact error probability, indicating the efficiency of the asymptotic BEP expressions.

Fig. 9 shows the asymptotic SNR penalty β_A as a function of $N_\mathrm{p}\varepsilon$ in Nakagami-m fading for several values of m and N_d. Several important observations can be made from looking at these graphs. First, note that curves are clustered according to the m parameter, with better performance (lower penalty) occurring for more benign environments, $m > 1$. More importantly, there is a surprising lack of dependence on N_d, if any. In particular, for $m > 1$ increasing N_d increases the SNR penalty slightly. However, for the case where $1/2 \le m < 1$, the effect is reversed; increasing N_d decreases the penalty. In all the cases investigated, the difference in SNR penalties between $N_\mathrm{d} = 1$ and $N_\mathrm{d} = 8$ does not exceed 0.2 dB. For the case of Rayleigh or Ricean fading, where $m = 1$, changes in N_d have no effect on the SNR penalty. This can be seen from the $m = 1$ curve in Fig. 9, where the curves line up for all N_d. These results are surprising because one could expect that, as the number of diversity branches increases, the error due to practical channel estimation would also increase, thereby

[8] Figs. 7 and 8 show the BEP for error rates as low as 10^{-10} only to illustrate the asymptotic behavior and to further provide numerical confirmation that the practical channel estimation scheme preserves the diversity order; these extremely low BEPs are not practical, especially for wireless mobile communications.

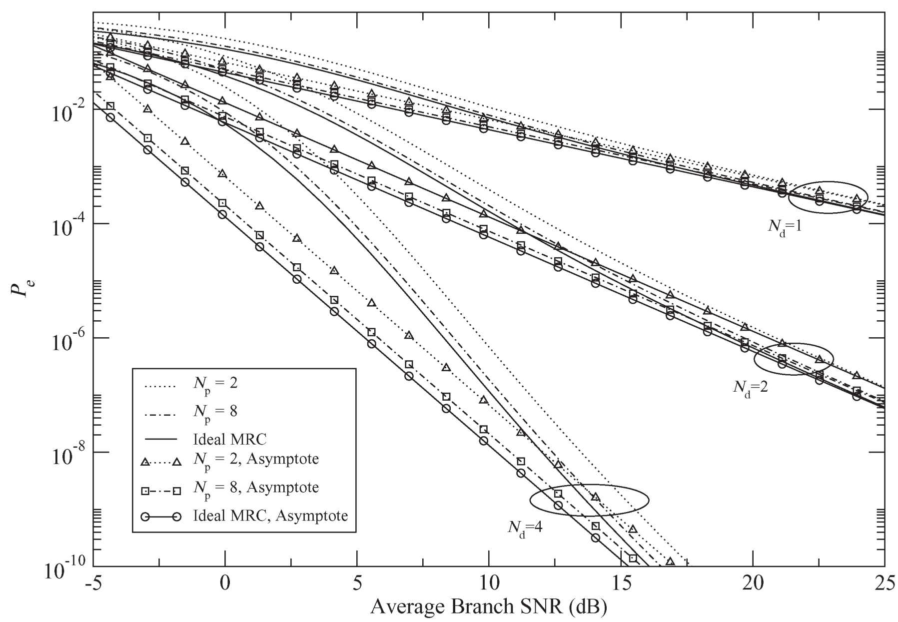

Fig. 8. Comparison of exact performance with asymptotic performance of BPSK in i.i.d. Ricean fading with $\kappa = 5$ dB, for various N_d, N_p.

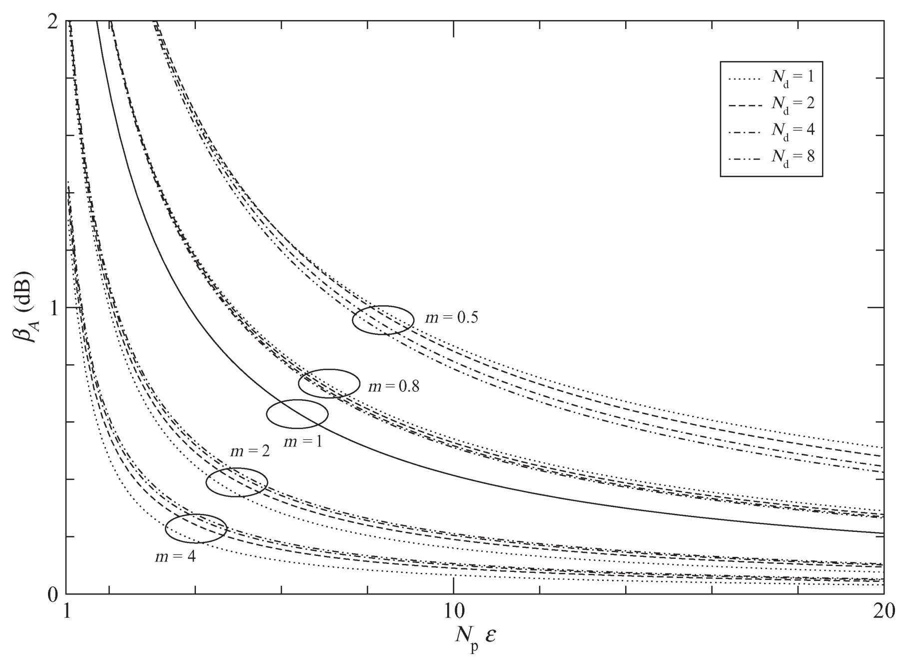

Fig. 9. SNR penalty as a function of $N_\mathrm{p}\varepsilon$, for various N_d and m.

incurring a larger SNR penalty. These results show that this is not the case.

VI. Conclusion

We have developed a general framework for evaluating the exact BEP of BPSK in N_d-branch diversity systems utilizing practical channel estimation. Our methodology, requiring only the evaluation of a single integral with finite limits, is applicable to channels with arbitrary distribution, including correlated fading, provided that the norm square of the channel-gain vector can be characterized by an MGF. We have shown that the pilot symbol estimation technique, appropriate for digital communication systems, preserves the diversity order of an N_d-branch diversity system; in contrast to the results of [15], [16], and [19]–[21], where the BEP was analyzed for fixed values of correlation. The SNR penalty, arising from practical channel estimation, was quantified. It was shown that the penalty has little dependence on the diversity order of the system.

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