MMSE Reception and Successive Interference Cancellation for MIMO Systems With High Spectral Efficiency

Alberto Zanella, *Member, IEEE*, Marco Chiani, *Senior Member, IEEE*, and Moe Z. Win, *Fellow, IEEE*

***Abstract*—In this paper, we investigate the performance in terms of symbol error probability (SEP) of multiple-input-multiple-output (MIMO) systems with high spectral efficiency. In particular, we consider the coherent detection of M-PSK signals in a flat Rayleigh-fading environment. We focus on spectrally efficient MIMO systems where, after serial-to-parallel conversion, several substreams of symbols are simultaneously transmitted by using an antenna array, thereby increasing the spectral efficiency. The reception is based on linear minimum mean-square-error (MMSE) combining, eventually followed by successive interference cancellation. Exact and approximate expressions are derived for an arbitrary number of transmitting and receiving antenna elements. Simulation results confirm the validity of our analytical methodology.**

***Index Terms*—Error propagation, minimum mean-square-error (MMSE) methods, multiple-input-multiple-output (MIMO) systems, Rayleigh channels, successive interference cancellation.**

I. Introduction

OVER the last several decades, multiple antennas have been used to combat fast fading. The increase in diversity order provided by diversity techniques enable robust communications in a fading environment [1], [2]. More recently, it has been recognized that the capacity of wireless communication links is increased by using multiple antennas both at the transmitter and the receiver [3], [4]. Toward achieving these capacities, a promising transmission system, called Diagonal-Bell Laboratories Layered Space-Time (D-BLAST), has been proposed in [3]. This scheme is able to provide a high spectral efficiency in a rich and quasi-static scattering environment. Owing to the large computational complexity required for this scheme, a simplified version, called Vertical BLAST (V-BLAST) has been proposed in [5]. A BLAST scheme is primarily based on the following three steps: 1) interference nulling to reduce the effect of the other (interfering) signals on the desired one; 2) ordering to select the substream with the largest signal-to-noise ratio (SNR); and 3) successive interference cancellation (SIC). Moreover, it was observed also in [3] that a minimum mean-square-error (MMSE) criterion can be used instead of interference nulling (zero forcing) to mitigate both interference and thermal noise.

In the literature, some papers (see, for example, [4], [6]–[9]) propose an analytical evaluation of the capacity of MIMO systems; some others address simulation of MIMO systems in frequency flat or selective fading channels [10]–[12], while [13], [14] investigate the capability of V-BLAST systems with maximum likelihood detection to reduce interference and thermal noise contributions. In this paper, we investigate the performance of multiple-input-multiple-output (MIMO) systems with high spectral efficiency. In particular, we consider the symbol error probability (SEP) for coherent detection of M-ary phase shift keying (PSK) signals in a flat Rayleigh-fading environment.

We start from the analytical framework developed in [15] and [16] for optimum combining (OC) of signals in multiantenna systems in the presence of co-channel interferers and thermal noise. This framework enables us to investigate the performance of MIMO-MMSE systems in a flat Rayleigh-fading environment. We generalize this methodology to derive the SEP for MIMO-MMSE followed by SIC. We refer to this system as MIMO-MMSE-SIC and first investigate its performance for the cases of no error propagation (EP). We then extend our study to include the effects of EP.

The paper is organized as follows. In Section II, we provide the system description and the basic equations for MIMO-MMSE receivers. In Section III, we give a new expression for analyzing optimum combining of signals, and derive the SEP of MIMO-MMSE. In Section IV, we present the performance analysis of MIMO-MMSE with SIC. Finally, in Section V, we show some numerical results, including the comparison with simulation, and conclusions are given in Section VI.

II. System Model for MIMO-MMSE

Throughout the paper, the superscript † denotes conjugation and transposition; vectors and matrices are indicated by bold, $|\mathbf{A}|$ and $\det \mathbf{A}$ denote the determinant of matrix $\mathbf{A}$, and $\{a_{i,j}\}_{i,j=1,\ldots,N}$ is an $N \times N$ matrix with elements

Manuscript received February 11, 2004; accepted April 7, 2004. The editor coordinating the review of this paper and approving it for publication is M. Shafi. The research was supported in part by Ministero dell'Istruzione, Università e della Ricerca Scientifica (MIUR) under the Virtual Immersive Communications (VICom) project (www.vicom-project.it), in part by the Institute of Advanced Study Natural Science and Technology Fellowship, in part by the Office of Naval Research Young Investigator Award N00014-03-1-0489, in part by the National Science Foundation under Grant ANI-0335256, and in part by the Charles Stark Draper Laboratory Robust Distributed Sensor Networks Program. This paper was presented in part at 2002 Conference on Information Sciences and Systems, Princeton University, NJ, March 2002.

A. Zanella and M. Chiani are with IEIIT-BO/CNR, DEIS, University of Bologna, 40136 Bologna, Italy (e-mail: azanella@deis.unibo.it; mchiani@deis.unibo.it).

M. Win is with the Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA 02139 USA (e-mail: moewin@mit.edu).

Digital Object Identifier 10.1109/TWC.2005.847103

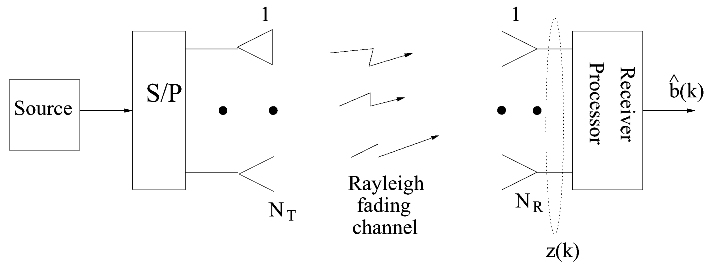

Fig. 1. Baseband model of a MIMO system.

$a_{i,j}, i,j = 1,\ldots,N$. The MIMO system investigated in this work is characterized by N_{T} transmitting and N_{R} receiving antennas (see Fig. 1); the original data stream is divided in N_{T} substreams, which are simultaneously transmitted by N_{T} parallel M-PSK modulators. The N_{R}-dimensional signal $\mathbf{z}(k)$ at the output of the receiving antennas at time k can be written as

$$\mathbf{z}(k) = \sqrt{E_{\mathrm{D}}}\mathbf{C}\mathbf{b}(k) + \mathbf{n}(k) \tag{1}$$

where E_{D} is the mean (over fading) received energy of the signal transmitted by each antenna, $\mathbf{b}(k)$ accounts for the transmitted symbols with $\mathbb{E}\{\mathbf{b}(k)\} = \mathbf{0}$ and $\mathbb{E}\{\mathbf{b}(k)\mathbf{b}(k)^{\dagger}\} = \mathbf{I}$, $\mathbf{n}(k)$ is the additive Gaussian noise vector with $\mathbb{E}\{\mathbf{n}(k)\cdot\mathbf{n}(k)^{\dagger}\} = N_0\mathbf{I}$, and $N_0/2$ is the two-sided thermal noise power spectral density per antenna element. The matrix $\mathbf{C}$ is the $(N_{\mathrm{R}} \times N_{\mathrm{T}})$ channel matrix

$$\mathbf{C} = \begin{bmatrix} | & | & & | \\ \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_{N_{\mathrm{T}}} \\ | & | & & | \end{bmatrix} \tag{2}$$

whose jth column consists of the propagation vector $\mathbf{c}_j$ corresponding to the jth transmitting antennas. As in [1], [3]–[7], we consider slow frequency flat fading with the elements of $\mathbf{C}$, modeled as independent identically distributed (i.i.d.) circular complex-valued Gaussian random variables (r.v.'s) having $\mathbb{E}\{c_{i,j}\} = 0$ and $\mathbb{E}\{|c_{i,j}|^2\} = 1$.

In a MIMO system based on linear combining, the received vector $\mathbf{z}(k)$ is combined with the matrix $\mathbf{W}$ to obtain the decision variables

$$\tilde{\mathbf{b}}(k) = \mathbf{W}^{\dagger}\cdot\mathbf{z}(k). \tag{3}$$

The choice of $\mathbf{W}$ minimizing the expected square-error (MMSE criterion) between the transmitted symbols and the decision variables is given by the following well-known result (see, e.g., [17, p. 438])

$$\mathbf{W}^{\dagger} = \sqrt{E_{\mathrm{D}}}\mathbf{B}\mathbf{C}^{\dagger}(E_{\mathrm{D}}\mathbf{C}\mathbf{B}\mathbf{C}^{\dagger} + N_0\mathbf{I})^{-1} \tag{4}$$

where $\mathbf{B} = \mathbb{E}\{\mathbf{b}(k)\mathbf{b}(k)^{\dagger}\}$. Using the hypothesis of independence among transmitted symbols, we have in our case $\mathbf{B} = \mathbf{I}$ and (4) becomes

$$\mathbf{W}^{\dagger} = \sqrt{E_{\mathrm{D}}}\mathbf{C}^{\dagger}\mathbf{R}^{-1} \tag{5}$$

where the covariance matrix $\mathbf{R}$ is given by[1]

$$\mathbf{R} = \mathbb{E}_{\mathbf{n},\mathbf{b}(k)}\{\mathbf{z}(k)\mathbf{z}(k)^{\dagger}\} = E_{\mathrm{D}}\mathbf{C}\mathbf{C}^{\dagger} + N_0\mathbf{I}. \tag{6}$$

In the following, MIMO systems with combining matrix $\mathbf{W}$ (5) have refereed to as MIMO-MMSE.

After linear MMSE reception, the vector $\tilde{\mathbf{b}}(k)$ containing the linear MMSE estimates of the transmitted symbols $\mathbf{b}(k)$ is further processed by a decision device to produce the estimated symbols $\hat{\mathbf{b}}(k)$. In its simplest form, the decision device is composed of a bank of parallel devices, one for each component of $\tilde{\mathbf{b}}(k)$. This can be also interpreted as a (vector) linear equalizer, where the aim is to reduce the "intersymbol" interference (ISI) due to the parallel transmission of independent symbols over the nonorthogonal radio channel, rather than the ISI among symbols transmitted at different time epoch as in single channel systems. More sophisticated suboptimum strategies can be designed, including successive interference cancellation that acts in an analogous way to decision feedback equalizers, and will be investigated in Section IV.

III. Performance Evaluation of MIMO-MMSE

In this section, we will derive the SEP performance of MIMO-MMSE receiver. The crux of the derivation is to observe that MIMO-MMSE reception is equivalent to a bank of parallel optimum combiners each with N_{R} antennas estimating the signal transmitted by one of the antenna, and treating signals from other remaining $N_{\mathrm{T}} - 1$ antennas as interferers. This fact, proven in Appendix A, enables us to obtain the SEP performance of MIMO-MMSE by leveraging the analytical framework developed for optimum combining. Recently [15], [16], and [18] consider coherent detection of M-PSK modulated signals using OC with N_{A} receiving antennas in the presence of N_{I} equal-power interferers, each with mean (over fast fading) received energy per symbol E_{I}, and thermal noise in a flat Rayleigh-fading environment.

The SEP expressing of [16, eq. (20)] requires the evaluation of nested $N_{\min}$-fold integrals, with $N_{\min} \triangleq \min\{N_{\mathrm{R}}, N_{\mathrm{T}}-1\}$, that it can be cumbersome for large $N_{\min}$. We extended this to reduce the computational complexity, as follows.

Theorem 1: The exact SEP expression for coherent detection of M-ary PSK with optimum combining, N_{R} receive, and $N_{\mathrm{T}} - 1$ equal-power co-channel interferers and thermal noise is

$$P_e = \frac{K}{\pi}\int_0^{\Theta} A(\theta)\det\mathcal{G}(\theta)\,d\theta \tag{7}$$

[1]We denote by $\mathbb{E}_X\{\cdot\}$ the expectation with respect to the r.v. X.

where

$$A(\theta) \triangleq \left[\frac{\sin^2\theta}{\sin^2\theta + c_{\rm MPSK}\frac{E_{\rm D}}{N_0}}\right]^{N_{\rm R}-N_{\rm min}} \quad (8)$$

with $\Theta \triangleq \pi(M-1)/M$, and $c_{\rm MPSK} = \sin^2(\pi/M)$. The matrix $\mathcal{G}(\theta) = \{G_{i+j}(\theta)\}_{i,j=0,\ldots,N_{\rm min}-1}$ is a Hankel matrix, with elements given by

$$G_{k-N_{\rm max}+N_{\rm min}}(\theta) = \zeta(\theta)^k e^{\zeta(\theta)} k! \Big[\zeta(\theta)(1+k) \times \Gamma(-1-k,\zeta(\theta)) + \frac{N_0}{E_{\rm I}}\Gamma(-k,\zeta(\theta))\Big] \quad (9)$$

where $\zeta(\theta) \triangleq (c_{\rm MPSK}E_{\rm D})/(E_{\rm I}\sin^2\theta) + N_0/E_{\rm I}$, and $\Gamma(a,z)$ is the incomplete Gamma function.

Proof: The exact expression for the SEP of a system with $N_{\rm A} = N_{\rm R}$ receiving antennas and $N_{\rm I} = N_{\rm T} - 1$ equal-power co-channel interferers is[2]

$$P_e = \mathbb{E}_{\tilde{\boldsymbol\lambda}}\{P_{e\,|\,\tilde{\boldsymbol\lambda}=\mathbf{x}}\} = \int_0^\infty \cdots \int_0^\infty \int_0^\infty P_{e\,|\,\tilde{\boldsymbol\lambda}}(\mathbf{x}) \times f_{\tilde{\boldsymbol\lambda}}(\mathbf{x})\, dx_1\, dx_2 \ldots dx_{N_{\rm min}} \quad (10)$$

where $P_{e\,|\,\tilde{\boldsymbol\lambda}=\mathbf{x}}$ is the error probability conditioned on a given realization $\mathbf{x} = [x_1,\ldots,x_{N_{\rm min}}]^T$ of the $N_{\rm min}$ nonzero unordered eigenvalues $\tilde{\boldsymbol\lambda} = [\tilde\lambda_1,\ldots,\tilde\lambda_{N_{\rm min}}]$ of a central Wishart matrix (see Appendix B), defined as

$$\mathcal{W}(N_{\rm min},N_{\rm max}) \triangleq \begin{cases} \mathbf{C}_j\mathbf{C}_j^\dagger, & \text{if } N_{\rm R} \le N_{\rm T}-1 \\ \mathbf{C}_j^\dagger\mathbf{C}_j & \text{, if } N_{\rm R} > N_{\rm T}-1 \end{cases} \quad (11)$$

where $N_{\rm max} = \max\{N_{\rm R}, N_{\rm T}-1\}$.

$P_{e\,|\,\tilde{\boldsymbol\lambda}=\mathbf{x}}$ is given by [16, eq. (18)]

$$P_{e\,|\,\tilde{\boldsymbol\lambda}}(\mathbf{x}) = \frac{1}{\pi}\int_0^\Theta A(\theta)\prod_{i=1}^{N_{\rm min}}\left[\frac{\sin^2\theta}{\sin^2\theta + \frac{c_{\rm MPSK}E_{\rm D}}{E_{\rm I}x_i+N_0}}\right] d\theta \quad (12)$$

and $f_{\tilde{\boldsymbol\lambda}}(\mathbf{x})$ is the joint probability density function (pdf) of $\tilde{\boldsymbol\lambda}$ given by (50) of Appendix B.

Substituting (50) in (10), we get

$$P_e = \frac{K}{\pi N_{\rm min}!}\int_0^\Theta\int_0^\infty\cdots\int_0^\infty A(\theta)|\mathbf{V}_1(\mathbf{x})|^2 \times \prod_{i=1}^{N_{\rm min}}\left[\left(\frac{\sin^2\theta}{\sin^2\theta+\frac{E_{\rm D}c_{\rm MPSK}}{E_{\rm I}x_i+N_0}}\right) \times e^{-x_i}x_i^{N_{\rm max}-N_{\rm min}}\right] dx_1\ldots dx_{N_{\rm min}}\, d\theta \quad (13)$$

[2]The expression (10) was obtained in [16, eq. (20)] with the exception of different integration limits. This difference is due to the fact that unordered nonzero eigenvalues of the covariance matrix $\tilde{\mathbf{R}}$ of [16] are used here in (10). In this paper, the covariance matrix $\tilde{\mathbf{R}}$ of [16] is denoted by $\tilde{\mathbf{R}}_j$ (see Appendix A).

where $\mathbf{V}_1(\mathbf{x})$ is a Vandermonde matrix (see Appendix B). The expression (13) can be simplified using the following Lemma, whose proof is given in [9].

Lemma 1: Given two arbitrary $N \times N$ matrices $\boldsymbol\Phi(\mathbf{x})$ and $\boldsymbol\Psi(\mathbf{x})$ with ijth elements $\Phi_i(x_j)$ and $\Psi_i(x_j)$, and an arbitrary function $\xi(\cdot)$, the following identity holds:

$$\int\cdots\int_{\mathcal{D}}|\boldsymbol\Phi(\mathbf{x})|\cdot|\boldsymbol\Psi(\mathbf{x})|\prod_{i=1}^N\xi(x_i)\,d\mathbf{x} = N!\det\left(\left\{\int_a^b\Phi_i(x)\Psi_j(x)\xi(x)\,dx\right\}_{i,j=1\ldots,N}\right) \quad (14)$$

where the multiple integral is over the domain $\mathcal{D} = \{a \le x_1 \le b, a \le x_2 \le b, \ldots, a \le x_N \le b\}$ and $d\mathbf{x} = dx_1\, dx_2 \ldots dx_N$.

Using Lemma 1 with $a = 0, b \to \infty$, $N = N_{\rm min}$, $\boldsymbol\Phi(\mathbf{x}) = \mathbf{V}_1(\mathbf{x})$, $\boldsymbol\Psi(\mathbf{x}) = \mathbf{V}_1(\mathbf{x})$, and

$$\xi(x) = \left[\left(\frac{\sin^2\theta}{\sin^2\theta+\frac{E_{\rm D}c_{\rm MPSK}}{E_{\rm I}x+N_0}}\right)e^{-x}x^{N_{\rm max}-N_{\rm min}}\right]$$

we get

$$P_e = \frac{K}{\pi}\int_0^\Theta \det\left(\left\{\int_0^\infty e^{-x}x^{N_{\rm max}-N_{\rm min}+i+j-2} \times \left(\frac{x+\frac{N_0}{E_{\rm I}}}{x+\frac{N_0}{E_{\rm I}}+\frac{E_{\rm D}c_{\rm MPSK}}{E_{\rm I}\sin^2\theta}}\right)dx\right\}_{i,j=1,\ldots,N_{\rm min}}\right)d\theta. \quad (15)$$

Finally, using the following identity:

$$\int_0^\infty e^{-x}x^n\frac{x+a}{x+b}\,dx = b^n e^b n! \times[b(1+n)\Gamma(-1-n,b)+a\Gamma(-n,b)] \quad (16)$$

valid for $n > -1$ and $\arg\{b\} \ne \pi$, we obtain (7). ■

Theorem 1 provides a concise SEP expression and is amenable for efficient evaluation involving only a integral single with finite integration limit.

As shown in Appendix A, for MIMO systems we can think of the linear MMSE combiner as equivalent to a bank of parallel optimum (or MMSE) combiners, each considering one of the signals transmitted by an antenna as the desired signal and the remaining $N_{\rm T}-1$ as interferers, with $E_{\rm D} = E_{\rm I}$ due to the assumption of uniform mean power over the transmitting antennas.

When the decision device following the linear combiner is a bank of independent slicers, the SEP is the same for all layers and is given by (7) together with (8) and (9). In the following, the SEP of a MIMO-MMSE system with $N_{\rm T}$ transmitting and $N_{\rm R}$ receiving antennas is denoted as

$$P_{e,\rm MMSE}\left(N_{\rm R}, N_{\rm T}, \frac{E_{\rm D}}{N_0}\right) \quad (17)$$

where $(E_{\rm D})/(N_0)$ is the receive symbol SNR per transmitting antenna.

In the next section, the performance of various decision devices following the MMSE linear combiner is investigated.

IV. MIMO-MMSE With SIC

The practical receiver structure suggested originally in [3] includes a linear combiner and successive interference cancellation. Although a linear MMSE combiner is expected to perform better than zero-forcing combiner [3]–[5], the latter is usually investigated in the literature since it is easier to analyze. Here, we derive simple expressions for the performance of MIMO system with linear MMSE combiner followed by SIC and denoted by MIMO-MMSE-SIC.

We consider a low-complexity SIC algorithm in which one of the linear MMSE combiner outputs is chosen, and the corresponding transmitted symbol is estimated by a slicer. The contribution of the signal due to this detected symbol is then reconstructed and cancelled from the received vector. This same procedure is repeated for all remaining symbols.

We note that the performance of MIMO-MMSE-SIC can be improved by a proper ordering of the symbols to be detected on the basis of the instantaneous channel state; the evaluation of its performance is beyond the scope of the current paper.

It is well known that detection with decision feedback suffers from EP, that is, the cancellation of an erroneously detected symbol increases the power of the interfering terms and can cause significant performance degradation [19], [20]. The same phenomenon is present in MIMO receivers employing SIC. In the next subsections, we analyze the performance of MIMO-MMSE-SIC for the cases of without EP (NEP) as well as with EP.

A. *Performance of MIMO-MMSE-SIC Without EP*

Equation (17) provides the starting point for evaluating the performance of MIMO-MMSE-SIC with arbitrary choice of order in the symbol detection. Without loss of generality, in the following it will be assumed that in the ith step we detect the ith element $b_i(k)$ of $\mathbf{b}(k)$. It is easy to show that, with SIC, the SEP can be derived by using the following:

$$P_{e,\mathrm{MMSE-SIC}} = \frac{1}{N_{\mathrm{T}}} \sum_{i=1}^{N_{\mathrm{T}}} P_{e_i} \tag{18}$$

where P_{e_i} represents the probability of making an error in the detection of the ith symbol. To derive P_{e_i}, let us define $\mathbf{z}_{[i]}(k)$ as the received vector after the cancellation of the previously detected $(i-1)$ symbols, so that $\mathbf{z}_{[1]}(k) = \mathbf{z}(k)$.[3] In the absence of EP, we can write

$$\mathbf{z}_{[2]} = \mathbf{z} - \sqrt{E_{\mathrm{D}}}\mathbf{c}_1 b_1 = \sqrt{E_{\mathrm{D}}} \sum_{i=2}^{N_{\mathrm{T}}} \mathbf{c}_i b_i + \mathbf{n} \tag{19}$$

where $\mathbf{c}_i$ is the propagation vector corresponding to b_i. In general

$$\mathbf{z}_{[i]} = \sqrt{E_{\mathrm{D}}}\mathbf{C}_{[i]}\mathbf{b}_{[i]} + \mathbf{n} \tag{20}$$

[3]The index (k) is omitted in the rest of the paper for brevity.

where $\mathbf{b}_{[i]}$ is the vector of the remaining $N_{\mathrm{T}} - i + 1$ undetected symbols and $\mathbf{C}_{[i]}$ represents the $N_{\mathrm{R}} \times (N_{\mathrm{T}} - i + 1)$ channel matrix without the propagation vectors corresponding to the $(i-1)$ estimated symbols. Equation (20) shows that $\mathbf{z}_{[i]}$ can be thought of as the received vector of a MIMO-MMSE system with N_{R} receiving antennas and $N_{\mathrm{T}} - i + 1$ transmit antennas. Hence, P_{e_i} is equal to $P_{e,\mathrm{MMSE}}(N_{\mathrm{R}}, N_{\mathrm{T}} - i + 1, E_{\mathrm{D}}/N_0)$, and (18) becomes

$$P_{e,\mathrm{MMSE-SIC}} = \frac{1}{N_{\mathrm{T}}} \sum_{i=1}^{N_{\mathrm{T}}} P_{e,\mathrm{MMSE}} \left(N_{\mathrm{R}}, N_{\mathrm{T}} - i + 1, \frac{E_{\mathrm{D}}}{N_0} \right). \tag{21}$$

B. *Performance of MIMO-MMSE-SIC With EP*

Note that (18) holds even in the presence of EP, provided that the probabilities P_{e_i} take into account the effects of EP. Unfortunately, the determination of the exact expressions for P_{e_i} is difficult. Here, we present a simple approach to estimate these probabilities, which are shown to be very accurate in the numerical results section.

By using the total probability theorem, we can write

$$P_{e_i} = \sum_{j=0}^{N_i - 1} \mathbb{P}\left\{ e_i \,\middle|\, E_j^{(i)} \right\} \cdot \mathbb{P}\left\{ E_j^{(i)} \right\} \tag{22}$$

where the $N_i = 2^{i-1}$ mutually exclusive events $E_j^{(i)}$, with $\mathbb{P}\{\bigcup E_j^{(i)}\} = 1$, regarding the $(i-1)$ previous symbols decisions. $\mathbb{P}\{e_i \,|\, E_j^{(i)}\}$ is the probability of making an error in the detection of the ith symbol conditioned on the event $E_j^{(i)}$. Each event $E_j^{(i)}$ can be associated with a $(i-1)$-dimensional vector $\mathbf{s}_j^{(i)}$, with element $s_{j,m}^{(i)}$ equal to zero if the symbol at the step m has been correctly detected, one otherwise. For example $\mathbf{s}_j^{(i)} = [0, 1, 1, 0, \ldots, 1]^T$ represents the event that the first symbol has been correctly detected, the second has been incorrectly detected, and so on. It is convenient for what follows to assume that $\mathbf{s}_j^{(i)}$ is a $(i-1)$-dimensional vector containing the binary representation of the number j. To better understand our derivation of $\mathbb{P}\{e_i \,|\, E_j^{(i)}\}$, let us consider a simple example with $i = 2$. In this case, we have two sequences $\mathbf{s}_0^{(2)} = [0]$, and $\mathbf{s}_1^{(2)} = [1]$, associated to the events $E_0^{(2)}$ and $E_1^{(2)}$, respectively. $\mathbb{P}\{e_2 \,|\, E_0^{(2)}\}$ represents the error probability for the second symbol conditioned on the event that the first symbol has been correctly detected. Similarly $\mathbb{P}\{e_2 \,|\, E_1^{(2)}\}$ represents the error probability for the second symbol conditioned on the event that the first symbol has been erroneously detected. While $\mathbb{P}\{e_2 \,|\, E_0^{(2)}\}$ can be easily derived by using the results of Section IV-A, the evaluation of $\mathbb{P}\{e_2 \,|\, E_1^{(2)}\}$ is much more involved. To derive it, let us consider the received vector $\mathbf{z}_{[2]}$

$$\begin{aligned} \mathbf{z}_{[2]} &= \mathbf{z} - \sqrt{E_{\mathrm{D}}}\mathbf{c}_1 \hat{b}_1 \\ &= \sqrt{E_{\mathrm{D}}}\mathbf{c}_1(b_1 - \hat{b}_1) + \sqrt{E_{\mathrm{D}}} \sum_{i=2}^{N_{\mathrm{T}}} \mathbf{c}_i b_i + \mathbf{n} \\ &= \sqrt{E_{\mathrm{D}}}\mathbf{C}_{[2]}\mathbf{b}_{[2]} + \underbrace{\sqrt{E_{\mathrm{D}}}\mathbf{c}_1(b_1 - \hat{b}_1) + \mathbf{n}}_{=\mathbf{n}_{\mathrm{eq}}} \end{aligned} \tag{23}$$

where $\hat{b}_1$ is the erroneous decision regarding b_1. The comparison of (19) and (23) shows that the error on the previous symbol results in an additional disturbance, which can cause severe performance degradation. To estimate this effect, we approximate the term $\mathbf{n}_{\mathrm{eq}}$ as a Gaussian r.v. with $\mathbb{E}\{\mathbf{n}_{\mathrm{eq}}\} = \mathbf{0}$ and $\mathbb{E}\{\mathbf{n}_{\mathrm{eq}}\mathbf{n}_{\mathrm{eq}}^{\dagger}\} = (E_{\mathrm{D}}\mathbb{E}\{|b_1 - \hat{b}_1|^2\} + N_0)\mathbf{I}$, where expectations are with respect to thermal noise, symbols, and propagation vectors. It will be apparent in the Section V that this Gaussian model is adequate for obtaining the SEP performance of MIMO systems. In the high-SNR regime, most of the errors will be such that $\hat{b}_1$ is one of the neighboring symbols of b_1. In this case, $|b_1 - \hat{b}_1|^2 = 4\sin^2(\pi/M) \triangleq d_{\mathrm{MPSK}}$, and hence

$$\mathbb{E}\{\mathbf{n}_{\mathrm{eq}}\mathbf{n}_{\mathrm{eq}}^{\dagger}\} = (E_{\mathrm{D}}d_{\mathrm{MPSK}} + N_0)\mathbf{I}. \tag{24}$$

With these approximations, $\mathbf{z}_{[2]}$ in (23) is equivalent to the received vector of a MIMO-MMSE system with $N_{\mathrm{T}}-1$ (since one symbol has been already detected) transmitting antennas each giving an equivalent SNR per receiving antenna equal to

$$\frac{E_{\mathrm{D}}}{E_{\mathrm{D}}d_{\mathrm{MPSK}} + N_0} = \frac{1}{\frac{1}{E_{\mathrm{D}}/N_0} + d_{\mathrm{MPSK}}} \tag{25}$$

and, therefore

$$\mathbb{P}\left\{e_2 \,\middle|\, E_1^{(2)}\right\} \approx P_{e,\mathrm{MMSE}}\left(N_{\mathrm{R}}, N_{\mathrm{T}}-1, \frac{1}{\frac{1}{E_{\mathrm{D}}/N_0} + d_{\mathrm{MPSK}}}\right). \tag{26}$$

Equation (26) can be generalized for arbitrary i and $E_j^{(i)}$. In fact, if the detection of the previous $(i-1)$ symbols has caused n_e errors, the received vector $\mathbf{z}_{[i]}$ becomes

$$\begin{aligned}\mathbf{z}_{[i]} &= \sqrt{E_{\mathrm{D}}}\mathbf{C}_{[i]}\mathbf{b}_{[i]} + \sqrt{E_{\mathrm{D}}}\sum_{q\in\{\mathcal{A}_{n_e}\}}\mathbf{c}_q(b_q - \hat{b}_q) + \mathbf{n} \\ &= \sqrt{E_{\mathrm{D}}}\mathbf{C}_{[i]}\mathbf{b}_{[i]} + \mathbf{n}_{\mathrm{eq}}\end{aligned} \tag{27}$$

where $\mathcal{A}_{n_e}$ denotes the set of indexes of the n_e erroneously decided symbols. Since the transmitted symbols are statistically independent, we again have $\mathbb{E}\{\mathbf{n}_{\mathrm{eq}}\} = \mathbf{0}$ and $\mathbb{E}\{\mathbf{n}_{\mathrm{eq}}\mathbf{n}_{\mathrm{eq}}^{\dagger}\} = (n_e E_{\mathrm{D}}d_{\mathrm{MPSK}} + N_0)\mathbf{I}$. Finally, the term $P_{e\,|\,E_j}$ can be written as

$$\begin{aligned}\mathbb{P}\left\{e_i \,\middle|\, E_j^{(i)}\right\} &\approx P_{e,\mathrm{MMSE}} \\ &\times \left(N_{\mathrm{R}}, N_{\mathrm{T}}-(i-1), \frac{1}{\frac{1}{E_{\mathrm{D}}/N_0} + \eta_j^{(i-1)}d_{\mathrm{MPSK}}}\right)\end{aligned} \tag{28}$$

where $\eta_j^{(m)}$ indicates the number of ones (wrong symbols) in the first m positions of the vector $\mathbf{s}_j^{(i)}$.

Now, let us consider the evaluation of $\mathbb{P}\{E_j^{(i)}\}$. This can be written as

$$\mathbb{P}\left\{E_j^{(i)}\right\} = \mathbb{P}\left\{\mathbf{s}_j^{(i)}\right\} = \mathbb{P}\left\{\cap_{n=1}^{i-1} s_{j,n}^{(i)}\right\}. \tag{29}$$

By using the well-known relation $\mathbb{P}\{\cap_{n=1}^{N} A_n\} = \prod_{n=1}^{N}\mathbb{P}\{A_n \mid \cap_{m=1}^{n-1} A_m\}$, we obtain

$$\begin{aligned}\mathbb{P}\left\{\cap_{n=1}^{i-1} s_{j,n}^{(i)}\right\} &= \prod_{n=1}^{i-1}\mathbb{P}\left\{s_{j,n}^{(i)} \cap_{m=1}^{n-1} s_{j,m}^{(i)}\right\} \\ &= \mathbb{P}\left\{s_{j,1}^{(i)}\right\}\mathbb{P}\left\{s_{j,2}^{(i)} \,\middle|\, s_{j,1}^{(i)}\right\}\cdots \\ &\quad \mathbb{P}\left\{s_{j,(i-1)}^{(i)} \,\middle|\, \cap_{m=1}^{i-2} s_{j,m}^{(i)}\right\}.\end{aligned} \tag{30}$$

We now consider the term $\mathbb{P}\{s_{j,(i-1)}^{(i)} \mid \cap_{m=1}^{i-2} s_{j,m}^{(i)}\}$: it represents the probability of having an error $s_{j,(i-1)}^{(i)} = 1$, or a correct decision $s_{j,(i-1)}^{(i)} = 0$, in the detection of the $(i-1)$th symbol conditioned on the detection of the first $(i-2)$ symbols. This problem can be solved by means of the previous model used to evaluate $\mathbb{P}\{e_i \mid E_j^{(i)}\}$, giving (31), located at the bottom of the page. The particular structure of (31) allows us to write $\mathbb{P}\{E_j^{(i)}\}$ as the product of $(i-1)$ conditioned probabilities, each conditioned on the result of the detection of the previous symbols that can be calculated by using an expression similar to (31)

$$\mathbb{P}\left\{E_j^{(i)}\right\} \approx \prod_{n=1}^{i-1}\mathcal{P}\left(N_{\mathrm{R}}, N_{\mathrm{T}}-(n-1), \eta_j^{(n-1)}, s_{j,n}^{(i)}\right) \tag{32}$$

where

$$\mathcal{P}(x,y,z,w) \triangleq \begin{cases} 1 - P_{e,\mathrm{MMSE}}\left(x, y, \frac{1}{\frac{1}{E_{\mathrm{D}}/N_0} + z d_{\mathrm{MPSK}}}\right), & \text{if } w = 0 \\ P_{e,\mathrm{MMSE}}\left(x, y, \frac{1}{\frac{1}{E_{\mathrm{D}}/N_0} + z d_{\mathrm{MPSK}}}\right), & \text{if } w = 1. \end{cases} \tag{33}$$

The final result of the previous analysis gives the performance of MIMO-MMSE-SIC reception with the following simple expression:

$$\begin{aligned}P_{e,\mathrm{MMSE-SIC}} &\approx \frac{1}{N_{\mathrm{T}}}\sum_{i=1}^{N_{\mathrm{T}}}\sum_{j=0}^{N_i-1}\left\{\mathcal{P}\left(N_{\mathrm{R}}, N_{\mathrm{T}}-(i-1), \eta_j^{(i-1)}, 1\right)\right. \\ &\left.\times \prod_{n=1}^{i-1}\mathcal{P}\left(N_{\mathrm{R}}, N_{\mathrm{T}}-(n-1), \eta_j^{(n-1)}, s_{j,n}^{(i)}\right)\right\}.\end{aligned} \tag{34}$$

Note that, for a given value of E_{D}/N_0, the number of terms $P_{e,\mathrm{MMSE}}(\cdot)$ to be calculated in (34) depends only on N_{T}, and

$$\mathbb{P}\left\{s_{j,(i-1)}^{(i)} \,\middle|\, \cap_{m=1}^{i-2} s_{j,m}^{(i)}\right\} \approx \begin{cases} 1 - P_{e,\mathrm{MMSE}}\left(N_{\mathrm{R}}, N_{\mathrm{T}}-(i-2), \frac{1}{\frac{1}{E_{\mathrm{D}}/N_0} + \eta_j^{(i-2)}d_{\mathrm{MPSK}}}\right), & \text{if } s_{j,(i-1)}^{(i)} = 0, \\ P_{e,\mathrm{MMSE}}\left(N_{\mathrm{R}}, N_{\mathrm{T}}-(i-2), \frac{1}{\frac{1}{E_{\mathrm{D}}/N_0} + \eta_j^{(i-2)}d_{\mathrm{MPSK}}}\right), & \text{if } s_{j,(i-1)}^{(i)} = 1. \end{cases} \tag{31}$$

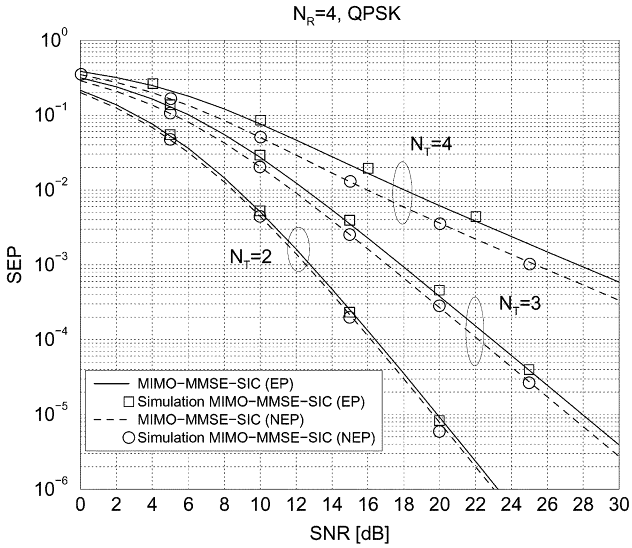

Fig. 2. Performance of MIMO-MMSE-SIC with EP and NEP for QPSK, $N_{\mathrm{R}}=4$, and various values of N_{T} ranging from 2 to 4.

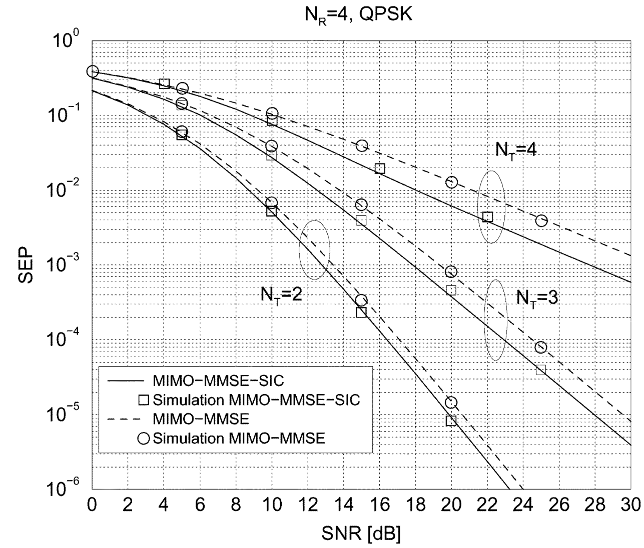

Fig. 3. Performance of MIMO-MMSE and MIMO-MMSE-SIC as a function of SNR for QPSK, $N_{\mathrm{R}}=4$, and various values of N_{T} ranging from 2 to 4.

it is given by $\sum_{j=1}^{N_{\mathrm{T}}} j = N_{\mathrm{T}}(N_{\mathrm{T}}-1)/2$. The error probability $P_{e,\mathrm{MMSE}}(\cdot)$ can be derived by using the exact formula (7) together with (8) and (9), or one of the approximate expressions given in [16].

V. Numerical Results

Here, we show the comparison between MIMO-MMSE and MIMO-MMSE-SIC based on the analytical expressions obtained in previous sections. The performance is evaluated in terms of SEP plotted as a function of the total SNR per receiving antenna element and spectral efficiency S_{eff}, defined as $(N_{\mathrm{T}}E_{\mathrm{D}})/N_0$ and $N_{\mathrm{T}} \cdot \log_2 M$, respectively. To assess the validity of the proposed approximate formulas, the performance of MIMO-MMSE and MIMO-MMSE-SIC is compared with bit-level simulations, where over 100 millions of symbols were generated.

The effect of EP on the performance of MIMO-MMSE-SIC is shown in Fig. 2, for $N_{\mathrm{R}}=4$, quarternary phase shift keying (QPSK) modulation, and N_{T} ranging from 2 to 4. The figure shows that, for instance, the target SEP is 10^{-3} and $N_{\mathrm{T}}=N_{\mathrm{R}}=4$, the hypothesis of absence of EP gives an optimistic estimate of the required SNR by about 3 dB. Furthermore, when small values of N_{T} are considered, the number of cancellation steps decreases and the system is less sensitive to EP. In all the subsequent figures, EP is taken into account. As clearly shown in the Fig. 3, MIMO-MMSE-SIC outperforms MIMO-MMSE; moreover, the results confirm that our analytical results, including the effects of EP, are in excellent agreement with simulation results.

The validity of the proposed model can also be appreciated in Figs. 4 and 5, where the same curves of Fig. 3 are shown but now N_{R} is equal to 5 and 6, and different values of N_{T} are considered. Note that using a number of receiver antennas larger than the number of transmit antennas gives a large improvement due to the additional diversity gain available.

Fig. 6 shows the SEP as a function of the number of transmit antennas for $N_{\mathrm{R}}=6$ and QPSK modulation different values of

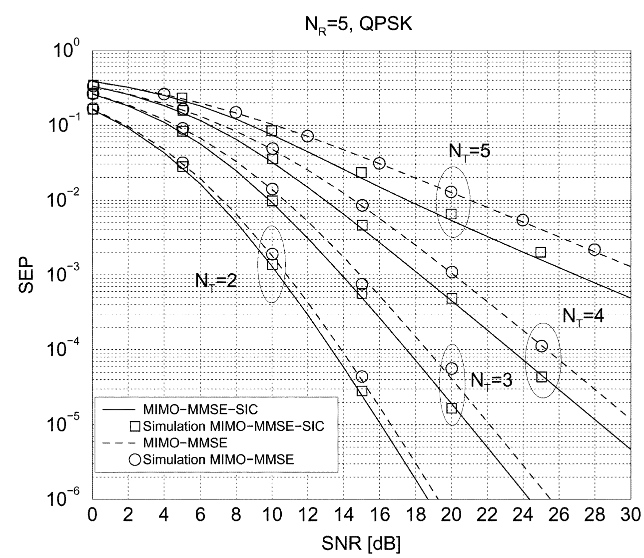

Fig. 4. Performance of MIMO-MMSE and MIMO-MMSE-SIC as a function of SNR for QPSK, $N_{\mathrm{R}}=5$, and various values of N_{T} ranging from 2 to 5.

SNR. The figure clearly shows how, as expected, increasing N_{T} degrades the SEP performance; on the other hand, increasing the number of transmit antennas provides an improvement in terms of spectral efficiency. In particular, if we fix the target SEP at 10^{-3}, we can achieve a spectral efficiency of 8 b/s/Hz ($N_{\mathrm{T}}=4$ and $M=4$) for values of SNR equal to 15 dB or larger.

To appreciate the exceptional spectral efficiencies provided by these practical systems, we show in Fig. 7 the spectral efficiency of MIMO-MMSE-SIC as a function of SNR for different values of N_{T} and N_{R} at target SEP of 10^{-3}. Fig. 7, obtained from the analytical expressions developed in previous sections, and including EP, shows that a spectral efficiency of 8 b/s/Hz can be achieved with a SNR of about 29 dB with $N_{\mathrm{R}}=4$ antennas; this value is reduced to about 15 dB if $N_{\mathrm{R}}=6$.

Finally, Fig. 8 is similar to Fig. 7 but with reference to the bit-error probability (BEP), here approximated as $\mathrm{SEP}/\log_2 M$

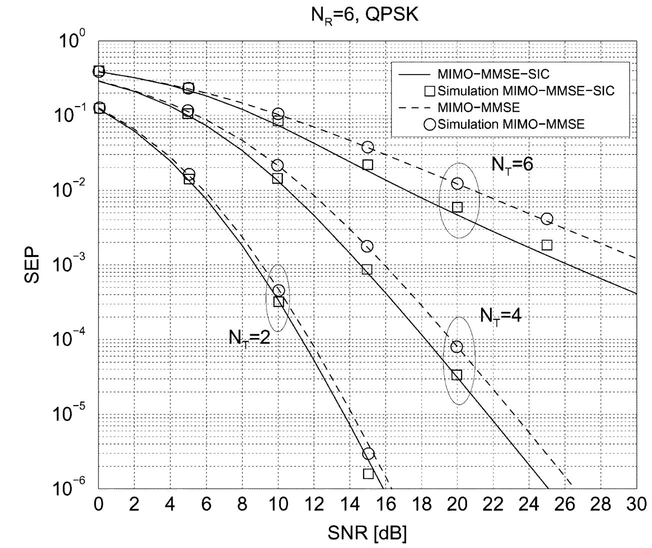

Fig. 5. Performance of MIMO-MMSE and MIMO-MMSE-SIC as a function of SNR for QPSK, $N_{\mathrm{R}} = 6$, and N_{T} equal to 2, 4, and 6.

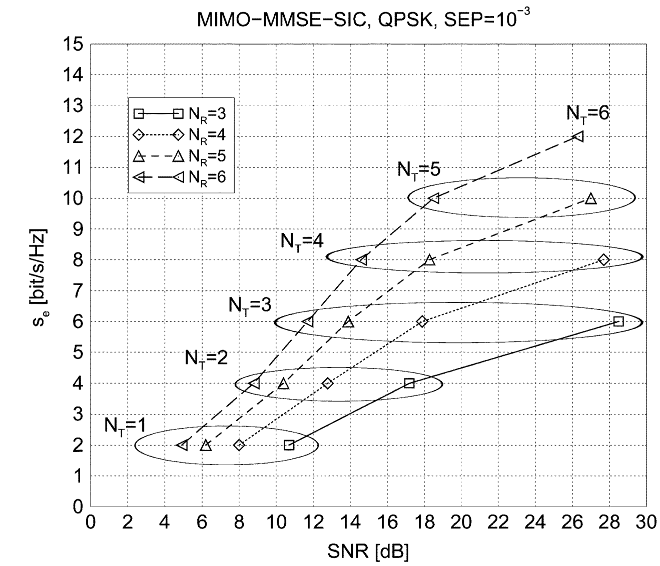

Fig. 7. Spectral efficiency of MIMO-MMSE-SIC as a function of SNR for QPSK and various values of $(N_{\mathrm{T}}, N_{\mathrm{R}})$ at target SEP of 10^{-3}.

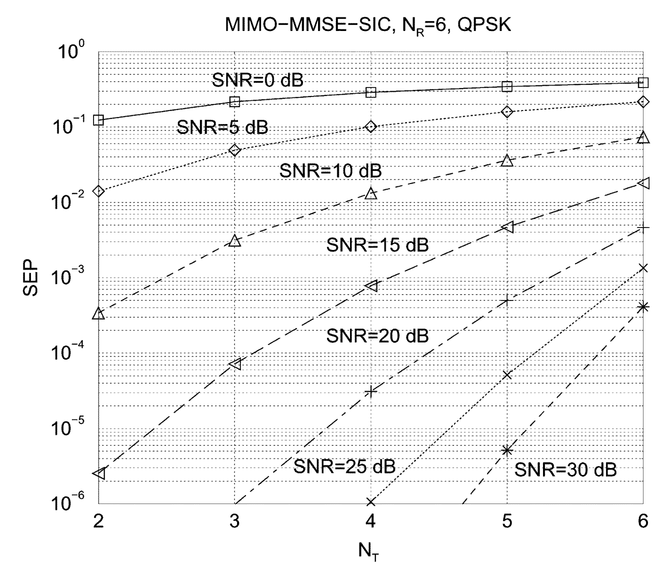

Fig. 6. Performance of MIMO-MMSE-SIC as a function of N_{T} for QPSK, $N_{\mathrm{R}} = 6$, and various values of SNR ranging from 0 to 30 dB.

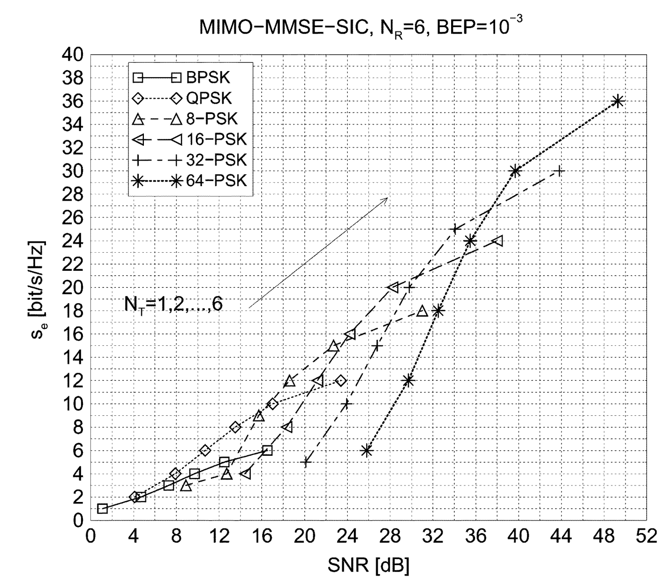

Fig. 8. Spectral efficiency of MIMO-MMSE-SIC as a function of SNR for $N_{\mathrm{R}} = 6$ and various modulation formats and N_{T} at target BEP of 10^{-3}.

under the hypothesis of Gray coding [22], for different modulation formats, $N_{\mathrm{R}} = 6$ receiver antennas and a target BEP of 10^{-3}. Symbols on a given curve indicate the number of transmit antennas (N_{T} ranges from 1 to 6). The crossing of the curves in the figure shows that for a given value of S_{eff} there exists an optimum modulation format with minimum required SNR. If we fix, for instance, $S_{\mathrm{eff}} = 16$ b/s/Hz, the minimum SNR is achieved with 16-PSK and four transmit antennas; the other modulation formats require either a larger value of SNR or provide a smaller spectral efficiency.

VI. Conclusion

In this paper, we have investigated the performance of high spectral efficiency MIMO systems with M-PSK signals in a flat Rayleigh-fading environment. We first proposed a methodology to evaluate the SEP for MIMO systems based on linear MMSE combining. Based on this methodology, we further derived the performance of MIMO-MMSE followed by successive interference cancellation. We then extended this to include the effect of EP. Our results are valid for arbitrary number of transmit and receive antennas and are confirmed by Monte Carlo simulations.

Appendix A
Equivalence Between Parallel Optimum and Joint MMSE Combining

We note that the problem of detecting the jth transmitted symbol in a MIMO system, in which case all others $N_{\mathrm{T}} - 1$ transmitted symbols can be thought of as interferers. Note that all the interfering signals are characterized by a mean (over the fast fading) received energy per symbol equal to $E_{\rm D}$. This scenario is equivalent to a multiantenna system with $N_{\rm A} = N_{\rm R}$ receiving antennas and $N_{\rm I} = N_{\rm T} - 1$ interfering signals, with a mean energy per symbol equal to $E_{\rm I} = E_{\rm D}$. It is well known that, with OC, the weights that maximize the output signal-to-interference-plus-noise ratio (SINR) are given by

$$\mathbf{w}_{{\rm OC},j} = \zeta_j \mathbf{R}_j^{-1}\mathbf{c}_j \tag{35}$$

where ζ_j is an arbitrary constant (which does not affect the array output SINR), $\mathbf{c}_j$ is the propagation vector corresponding to b_j, and $\mathbf{R}_j$ is the covariance matrix given by $\mathbf{R}_j = E_{\rm D}\tilde{\mathbf{R}}_j + N_0\mathbf{I}$ with $\tilde{\mathbf{R}}_j = \mathbf{C}_j\mathbf{C}_j^\dagger$, and

$$\mathbf{C}_j = \begin{bmatrix} | & | & & | & | & & | \\ \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_{j-1} & \mathbf{c}_{j+1} & \cdots & \mathbf{c}_{N_{\rm T}} \\ | & | & & | & | & & | \end{bmatrix}. \tag{36}$$

Here, we prove that (5) is equivalent to (35) and, therefore, the analytical frameworks developed in Section III can be applied for investigating MIMO with MMSE linear reception.

The jth row of the matrix $\mathbf{W}^\dagger$ in (5) that is given by

$$\mathbf{w}_j^\dagger = \sqrt{E_{\rm D}}\mathbf{c}_j^\dagger \mathbf{R}^{-1} \tag{37}$$

if we consider its conjugate transpose, we get

$$\mathbf{w}_j = \sqrt{E_{\rm D}}\mathbf{R}^{-1}\mathbf{c}_j. \tag{38}$$

where we used the property of the Hermitian matrices $(\mathbf{R}^{-1})^\dagger = \mathbf{R}^{-1}$. To show that (38) is equivalent to (35), we need the following theorem.

Theorem 2: Let $\mathbf{P} \in M_{m,n}$ and $\mathbf{P}_j \in M_{m,n-1}$ be given by

$$\mathbf{P} = \begin{bmatrix} | & | & & | \\ \mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_n \\ | & | & & | \end{bmatrix} \tag{39}$$

and

$$\mathbf{P}_j = \begin{bmatrix} | & | & & | & | & & | \\ \mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_{j-1} & {}_{j+1} & \cdots & \mathbf{p}_n \\ | & | & & | & | & & | \end{bmatrix} \tag{40}$$

where $M_{m,n}$ denotes the set of the $(m \times n)$ complex matrices. Let $\mathbf{Q}$ and $\mathbf{Q}_j$ be equal to $K_1\mathbf{P}\mathbf{P}^\dagger + K_2\mathbf{I}$ and $K_1\mathbf{P}_j\mathbf{P}_j^\dagger + K_2\mathbf{I}$, respectively, with scalars $K_1 \geq 0$ and $K_2 > 0$. Then, the complex vectors $\mathbf{v}_j = \mathbf{Q}^{-1}\mathbf{p}_j$ and $\mathbf{s}_j = \mathbf{Q}_j^{-1}\mathbf{p}_j$ are related as $\mathbf{v}_j = \gamma_j\mathbf{s}_j$ where $\gamma_j = 1/(1 + K_1\mathbf{p}_j^\dagger\mathbf{Q}_j^{-1}\mathbf{p}_j)$ is a real nonnegative number.

Proof: Note that for any given $\mathbf{P}$, the square matrices $\mathbf{Q}$ and $\mathbf{Q}_j$ have nonzero determinants and, hence, $\mathbf{Q}^{-1}$ and $\mathbf{Q}_j^{-1}$ exist. We can relate $\mathbf{v}_j$ and $\mathbf{s}_j$ as

$$\mathbf{v}_j = \mathbf{Q}^{-1}\mathbf{p}_j = \mathbf{Q}^{-1}(\mathbf{Q}_j\mathbf{s}_j). \tag{41}$$

Since that $\mathbf{Q} = \mathbf{Q}_j + K_1\mathbf{p}_j\mathbf{p}_j^\dagger$, we have

$$\begin{aligned}\mathbf{v}_j &= \mathbf{Q}^{-1}(\mathbf{Q} - K_1\mathbf{p}_j\mathbf{p}_j^\dagger)\mathbf{s}_j \\ &= \mathbf{s}_j - \mathbf{v}_j K_1 \mathbf{p}_j^\dagger \mathbf{Q}_j^{-1}\mathbf{p}_j \end{aligned} \tag{42}$$

where we have used the fact that $\mathbf{Q}^{-1}\mathbf{p}_j = \mathbf{v}_j$ and $\mathbf{s}_j = \mathbf{Q}_j^{-1}\mathbf{p}_j$. Therefore

$$\mathbf{v}_j = \frac{\mathbf{s}_j}{1 + K_1\mathbf{p}_j^\dagger\mathbf{Q}_j^{-1}\mathbf{p}_j} = \gamma_j\mathbf{s}_j \tag{43}$$

and, hence, the vector $\mathbf{v}_j$ is proportional to $\mathbf{s}_j$. Since $\mathbf{Q}_j$ is positive definite, $\mathbf{p}_j^\dagger\mathbf{Q}_j^{-1}\mathbf{p}_j > 0$ which implies that the proportionality constant γ_j is real positive. ■

If we now define $\mathbf{P} = \mathbf{C}, \mathbf{P}_j = \mathbf{C}_j, K_1 = E_{\rm D}, K_2 = N_0$, (38) can be rewritten using Theorem 2 as

$$\mathbf{w}_j = \frac{\sqrt{E_{\rm D}}\mathbf{R}_j^{-1}\mathbf{c}_j}{1 + E_{\rm D}\mathbf{c}_j^\dagger\mathbf{R}_j^{-1}\mathbf{c}_j} = \gamma_j\sqrt{E_{\rm D}}\mathbf{R}_j^{-1}\mathbf{c}_j \tag{44}$$

Note that (44) is in the form (35). This establishes the equivalence between (5) and (35) and, therefore, all the results for optimum combining aiming to maximize the SINR can be used to investigate the MIMO-MMSE systems.

APPENDIX B
DISTRIBUTION OF THE UNORDERED EIGENVALUES OF A WISHART MATRIX

Let us define the $(N_{\min} \times N_{\max})$, with $N_{\min} \leq N_{\max}$, complex matrix $\mathbf{A}$, with $\mathbf{\Sigma} = \mathbb{E}\{\mathbf{a}_j\mathbf{a}_j^\dagger\}\ \forall j$ and $\mathbb{E}\{\mathbf{a}_i\mathbf{a}_j^\dagger\} = \mathbf{0}$ for $i \neq j$, where $\mathbf{a}_j$ is the jth column vector of $\mathbf{A}$. If the elements of $\mathbf{A}$, a_{ij}, are complex values with real and imaginary part each belonging to a normal distribution $\mathcal{N}(0, 1/2)$, then the Hermitian matrix $\mathcal{W}(N_{\min}, N_{\max}) = \mathbf{A}\mathbf{A}^\dagger$ is called central Wishart. The distribution of the eigenvalues is studied in [22]. The joint pdf of the (real) ordered eigenvalues $\tilde{\lambda}_1 \geq \tilde{\lambda}_2 \geq \ldots \geq \tilde{\lambda}_{N_{\min}}$ is

$$\begin{aligned} f_{\tilde{\boldsymbol{\lambda}}}(x_1, \ldots, x_{N_{\min}}) &= K|\mathbf{\Sigma}|^{-N_{\max}}{}_0\tilde{F}_0(-\mathbf{\Sigma}^{-1}, \mathbf{W}) \\ &\quad \times |\mathbf{W}|^{N_{\max}-N_{\min}} \prod_{i<j}^{N_{\min}} (x_i - x_j)^2 \end{aligned} \tag{45}$$

where K is a normalizing constant given by

$$K = \frac{\pi^{N_{\min}(N_{\min}-1)}}{\tilde{\Gamma}_{N_{\min}}(N_{\max})\tilde{\Gamma}_{N_{\min}}(N_{\min})} \tag{46}$$

with

$$\tilde{\Gamma}_{N_{\min}}(n) = \pi^{N_{\min}(N_{\min}-1)/2} \prod_{i=1}^{N_{\min}} (n-i)!. \tag{47}$$

${}_0\tilde{F}_0(\mathbf{A},\mathbf{B})$ is known as hypergeometric function of Hermitian matrix arguments, whose definition is given in [22, (88)] in terms of series involving *zonal polynomials*. These polynomials are in general very difficult to manage.

In case of $\mathbf{\Sigma} = \mathbf{I}$, the joint pdf of the (real) ordered eigenvalues $\tilde{\lambda}_1 \geq \tilde{\lambda}_2 \cdots \geq \tilde{\lambda}_{N_{\min}}$ of $\mathcal{W}(N_{\min}, N_{\max})$ can be written as [22]

$$f_{\tilde{\boldsymbol{\lambda}}}(x_1,\ldots,x_{N_{\min}}) = K\prod_{i=1}^{N_{\min}} e^{-x_i} x_i^{N_{\max}-N_{\min}} \cdot \prod_{i<j}^{N_{\min}} (x_i - x_j)^2. \quad (48)$$

Denoting $\mathbf{x} = [x_1, x_2, \ldots, x_{N_{\min}}]$, the pdf in (48) can be written alternatively in terms of the Vandermonde matrix $\mathbf{V}_1(\mathbf{x}) = \{x_j^{i-1}\}_{i,j=1,\ldots,N_{\min}}$. Since $|\mathbf{V}_1(\mathbf{x})|^2 = \prod_{i<j}^{N_{\min}} (x_i - x_j)^2$, (48) becomes

$$f_{\tilde{\boldsymbol{\lambda}}}(x_1,\ldots,x_{N_{\min}}) = K|\mathbf{V}_1(\mathbf{x})|^2 \prod_{i=1}^{N_{\min}} e^{-x_i} x_i^{N_{\max}-N_{\min}}. \quad (49)$$

Starting from (49), the joint pdf of the unordered eigenvalues of $\mathcal{W}(N_{\min}, N_{\max})$ is easily written as

$$\frac{K}{N_{\min}!}|\mathbf{V}_1(\mathbf{x})|^2 \prod_{i=1}^{N_{\min}} e^{-x_i} x_i^{N_{\max}-N_{\min}}. \quad (50)$$

ACKNOWLEDGMENT

The authors wish to thank J. H. Winters, G. J. Foschini, and L. A. Shepp for helpful discussions.

REFERENCES

[1] J. H. Winters, "On the capacity of radio communication systems with diversity in Rayleigh-fading environment," *IEEE J. Sel. Areas Commun.*, vol. SAC-5, no. 5, pp. 871–878, Jun. 1987.

[2] M. Z. Win and J. H. Winters, "Virtual branch analysis of symbol error probability for hybrid selection/maximal-ratio combining in Rayleigh fading," *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1926–1934, Nov. 2001.

[3] G. J. Foschini, "Layered space-time architecture for wireless communication a fading environment when using multiple antennas," *Bell Labs. Tech. J.*, vol. 1, no. 2, pp. 41–59, Autumn 1996.

[4] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

[5] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communication employing multielement arrays," *IEEE Trans. Commun.*, vol. 17, no. 11, pp. 1841–1852, Nov. 1999.

[6] E. Telatar, "Capacity of multiantenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov.–Dec. 1999.

[7] A. F. Molisch, M. Z. Win, and J. H. Winters, "Capacity of MIMO systems with antenna selection," in *Proc. IEEE Int. Conf. Communications*, vol. 2, Helsinki, Finland, Jun. 2001, pp. 570–574.

[8] S. Loyka and G. Tsoulos, "Estimating MIMO system performance using the correlation matrix approach," *IEEE Commun. Lett.*, vol. 6, no. 1, pp. 19–21, Jan. 2002.

[9] M. Chiani, M. Z. Win, and A. Zanella, "On the capacity of spatially correlated MIMO Rayleigh-fading channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2363–2371, Oct. 2003.

[10] S. Catreux, P. F. Driessen, and L. J. Greenstein, "Simulation results for an interference-limited multiple-input multiple-output cellular system," *IEEE Commun. Lett.*, vol. 4, no. 11, pp. 334–336, Nov. 2000.

[11] N. Boubaker, K. B. Letaief, and R. D. Murch, "Performance of BLAST over frequency-selective wireless communication channels," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 196–199, Feb. 2002.

[12] A. Lozano and C. Papadias, "Layered space-time receivers for frequency-selective wireless channels," *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 65–73, Jan. 2002.

[13] W.-J. Choi, R. Negi, and J. M. Cioffi, "Combined ML and DFE decoding for the V-BLAST system," in *Proc. IEEE Int. Conf. Communications*, vol. 3, Jun. 2000, pp. 1243–1248.

[14] X. Zhu and R. D. Murch, "Performance analysis of maximum likelihood detection in a MIMO antenna system," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 187–191, Feb. 2002.

[15] M. Chiani, M. Z. Win, A. Zanella, and J. H. Winters, "Efficient evaluation of exact error probability for optimum combining of M-ary PSK signals," in *Proc. IEEE Global Telecomm. Conf.*, vol. 2, Taipei, Taiwan, R.O.C., Nov. 2002, pp. 1415–1419.

[16] M. Chiani, M. Z. Win, A. Zanella, R. K. Mallik, and J. H. Winters, "Bounds and approximations for optimum combining of signals in the presence of multiple co-channel interferers and thermal noise," *IEEE Trans. Commun.*, vol. 51, no. 2, pp. 296–307, Feb. 2003.

[17] R. N. McDonough and A. D. Whalen, *Detection of Signals in Noise*, 2nd ed. San Diego, CA: Academic, 1995.

[18] A. Zanella, M. Chiani, M. Z. Win, and J. H. Winters, "Symbol error probability of high spectral efficiency MIMO systems," in *Proc. Conf. Information Sciences and Systems*, Princeton, NJ, Mar. 2002.

[19] M. Chiani, "Introducing erasures in decision-feedback equalization to reduce error propagation," *IEEE Trans. Commun.*, vol. 45, no. 7, pp. 757–760, Jul. 1997.

[20] D. L. Duttweiler, J. E. Mazo, and G. Messerschmitt, "An upper bound on the error probability in decision-feedback equalizers," *IEEE Trans. Inf. Theory*, vol. IT-20, no. 4, pp. 490–497, Jul. 1974.

[21] J. G. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2001.

[22] A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *Ann. Math. Statist.*, vol. 35, pp. 475–501, 1964.

Alberto Zanella (S'99–M'00) was born in Ferrara, Italy, on December 23, 1971. He received the Dr. Ing. degree (with honors) in electronic engineering from the University of Ferrara, Ferrara, Italy, in 1996, and the Ph.D. degree in electronic and computer science from the University of Bologna, Bologna, Italy, in 2000.

In 2001, he joined the Consiglio Nazionale delle Ricerche-Centro di Studio per l'Informatica e i sistemi di Telecomunicazioni (CNR-CSITE), now a section of CNR-Istituto di Elettronica e di Ingegneria dell'Informazione e delle Telecomunicazioni (IEIIT), as a Researcher. Since 2001, he has also been a Professor of Electrical Communication (2001 and 2003) and Telecommunication Systems (2002) with the University of Bologna. Additionally, he has been involved in EUROCOST 273 activities since 2001. His research interests include cellular and mobile radio systems, WLANs, radio resource management (handover algorithms, power control, frequency hopping, and dynamic channel allocation), adaptive equalization, smart antennas, and multiple-input multiple-output systems.

Dr. Zanella is/was on the Technical Program Committee of the IEEE ICC 2002, GLOBECOM 2003, IEEE ICC 2004, and IEEE ICC 2005.

Marco Chiani (M'94–SM'02) was born in Rimini, Italy, on April 4, 1964. He received the Dr.lng. degree (with honors) in electronic engineering and the Ph.D. degree in electronic engineering and computer science from the University of Bologna, Bologna, Italy, in 1989 and 1993, respectively.

From 1994 he has been with the Dipartimento di Elettronica, Informatica e Sistemistica, University of Bologna, Cesena, Italy, where he is currently Professor and Chair for Telecommunications. His research interests include the areas of communications theory, coding, and wireless networks.

Dr. Chiani is the Editor for Wireless Communications for the IEEE TRANSACTIONS ON COMMUNICATIONS, and former Chair of the Radio Communications Committee, IEEE Communications Society. He was in the Technical Program Committee of the IEEE conferences Globecom 1997, IEEE International Conference on Communications (ICC) 1999, ICC 2001, ICC 2002, Globecom 2003, and Technical Program Co-Chair for the IEEE Wireless Communications Symposium of the ICC 2004.

Moe Z. Win (S'85–M'87–S'97–F'04) received the B.S. degree (*magna cum laude*) from Texas A&M University, College Station, in 1987 and the M.S. degree from the University of Southern California (USC), Los Angeles, in 1989, both in electrical engineering. As a Presidential Fellow at USC, he received both an M.S. degree in Applied mathematics and the Ph.D. degree in electrical engineering in 1998.

He is currently an Associate Professor at the Laboratory for Information and Decision Systems (LIDS), Massachusetts Institute of Technology. Prior to joining LIDS, he spent five years at AT&T Research Laboratories and seven years at the Jet Propulsion Laboratory. His main research interests are the application of mathematical and statistical theories to communication, detection, and estimation problems. Specific current research topics include measurement and modeling of time-varying channels, design and analysis of multiple antenna systems, ultra-wide bandwidth (UWB) communications systems, optical communications systems, and space communications systems.

Dr. Win has been involved actively in organizing and chairing sessions, and has served as a member of the Technical Program Committee in a number of international conferences, including the IEEE Communication Theory Symposium of ICC-2004 and the IEEE Conference on Ultra Wideband Systems and Technologies in 2002. He is also the current chair and former secretary for the Radio Communications Technical Committee of the IEEE Communications Society. He currently serves as Area Editor for Modulation and Signal Design and Editor for Wideband Wireless and Diversity, both for the IEEE TRANSACTIONS ON COMMUNICATIONS. He served as the Editor for Equalization and Diversity from July 1998 to June 2003 for the IEEE TRANSACTIONS ON COMMUNICATIONS, and as a Guest-Editor for the 2002 IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS Special Issue on Ultra-Wideband Radio in Multiaccess Wireless Communications. He received the International Telecommunications Innovation Award from Korea Electronics Technology Institute in 2002, a Young Investigator Award from the Office of Naval Research in 2003, and the IEEE Antennas and Propagation Society Sergei A. Schelkunoff Transactions Prize Paper Award in 2003. In 2004, he was named Young Aerospace Engineer of the Year by AIAA, received the Fulbright Foundation Senior Scholar Lecturing and Research Fellowship, the Institute of Advanced Study Natural Sciences and Technology Fellowship, the Outstanding International Collaboration Award from the Industrial Technology Research Institute of Taiwan, and the Presidential Early Career Award for Scientists and Engineers from the White House.