The Effect of Narrowband Interference on Wideband Wireless Communication Systems

Andrea Giorgetti, *Member, IEEE*, Marco Chiani, *Senior Member, IEEE*, and Moe Z. Win, *Fellow, IEEE*

Abstract—**This paper evaluates the performance of wideband communication systems in the presence of narrowband interference (NBI). In particular, we derive closed-form bit-error probability expressions for spread-spectrum systems by approximating narrowband interferers as independent asynchronous tone interferers. The scenarios considered include additive white Gaussian noise channels, flat-fading channels, and frequency-selective multipath fading channels. For multipath fading channels, we develop a new analytical framework based on perturbation theory to analyze the performance of a Rake receiver in Nakagami-m channels. Simulation results for NBI such as GSM and Bluetooth are in good agreement with our analytical results, showing the approach developed is useful for investigating the coexistence of ultrawide bandwidth systems with existing wireless systems.**

Index Terms—**Code-division multiple-access (CDMA) systems, direct sequence (DS), matched filter (MF), narrowband interference (NBI), Rake reception, time-hopping (TH), ultrawide bandwidth (UWB) systems.**

I. Introduction

THERE has been an emerging interest in transmission systems with large bandwidth for both commercial and military applications [1]–[13]. For example, ultrawide bandwidth (UWB) systems communicate with time-hopping (TH) or direct-sequence (DS) spread-spectrum (SS) signals using a train of extremely short pulses, thereby spreading the energy of the signal very thinly over several gigahertz [1]–[5].

The potential strength of UWB systems lies in its use of extremely wide transmission bandwidths, resulting in desirable capabilities, including: 1) accurate position location and ranging, and lack of significant multipath fading due to fine delay resolution; 2) multiple access due to wide transmission bandwidths; 3) covert communications due to low transmission power operation; and 4) possible easier material penetration due to low-frequency components.

The use of large transmission bandwidths, on the other hand, introduces new challenges. In particular, the successful deployment of UWB systems requires that they coexist and contend with a variety of interfering signals. For example, unlicensed commercial systems are currently envisioned to operate with low power spectral levels over already-populated frequency bands under United States Federal Communication Commission (FCC) Part 15 rules [14]. Intentional jammers are invariably present in many military scenarios, and UWB systems must be robust against jamming. Such systems must also minimize the probability of detection and interception by nonintended users. This again requires robust transmission with low power spectral levels in the presence of intentional jammers. Although the specific goals of commercial and military UWB systems may be different, it is apparent that a thorough performance analysis of these systems in the presence of narrowband (NB) interferers is essential for their efficient design and successful operation. This paper focuses on the analysis of the effect of NB interference (NBI) on wideband wireless communications systems.[1]

Previous work on performance analysis of transmission schemes in the presence of NB or tone interferers has been largely focused on additive white Gaussian noise (AWGN) channels. For the case of single tone interferer, the bit-error probability (BEP) of DS-SS systems is derived in [15] and [16] for AWGN channels, and the comparison of several multiple-access techniques is made in [17] by ignoring the effects of additive noise. For the case of multiple tone interferers, an upper bound on the BEP was derived using the Chernoff bound in [18] and [19] for AWGN channels. The performance of UWB TH-SS systems is analyzed without additive noise in the presence of a single Gaussian interferer in [13] and with multiple tone interferers by means of the Gaussian approximation in [20]. By assuming that multiple access and a single-tone interference are both Gaussian, their effect on UWB TH-SS is investigated in [21] for AWGN channels, and in [22] for multipath fading channels. The effects of global systems for mobile communications (GSM) and universal mobile telecommunications (UMTS)/wideband code-division multiple-access (WCDMA) systems on UWB DS-SS and TH-SS systems in AWGN channels is investigated via simulation in [5]–[7]. Methods for selecting parameters to improve the anti-jam capability of UWB radios in AWGN are developed in [23] and

Paper approved by D. I. Kim, the Editor for Spread-Spectrum Transmission and Access of the IEEE Communications Society. Manuscript received August 13, 2004; revised March 18, 2005. This work was supported in part by the Ministero dell'Istruzione, Università e della Ricerca Scientifica (MIUR) under the Virtual Immersive Communications (VICom) project, in part by the Institute of Advanced Study Natural Science and Technology Fellowship, in part by the Office of Naval Research under Young Investigator Award N00014-03-1-0489, in part by the National Science Foundation under Grant ANI-0335256, and in part by the Charles Stark Draper Endowment. This paper was presented in part at the IEEE Vehicular Technology Conference, Milan, Italy, May 2004, and in part at the IEEE International Conference on Communications, Paris, France, June 2004.

A. Giorgetti and M. Chiani are with IEIIT-BO/CNR, DEIS, University of Bologna, 40136 Bologna, Italy (e-mail: agiorgetti@deis.unibo.it; mchiani@deis.unibo.it).

M. Z. Win is with the Laboratory for Information and Decision Systems (LIDS), Massachusetts Institute of Technology, Cambridge, MA 02139 USA (e-mail: moewin@mit.edu).

Digital Object Identifier 10.1109/TCOMM.2005.860047

[1]The effects of wideband interference on the performance of NB systems are not considered.

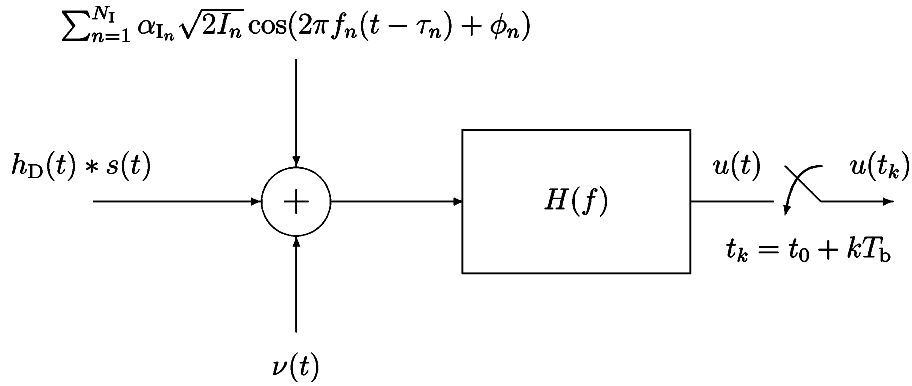

Fig. 1. System with multiple tone interferers and additive noise.

[24]. The use of notch filters to mitigate the effect of NBI is investigated in [25] and [26] for CDMA overlay, and in [27] for UWB systems. However, in the literature, there are no analytical results for both DS and TH systems with multiple tone interferers. Furthermore, BEP expressions for Rake reception in frequency-selective multipath fading channels in the presence of a tone interferer are absent from the literature.

In this paper, we analyze the performance of binary SS systems in the presence of NBI. In particular, we derive closed-form expressions for the BEP of a general binary coherent system with matched-filter (MF) reception. We approximate NB interferers as independent asynchronous tone interferers with arbitrary frequencies. A new analytical framework based on perturbation theory is developed to analyze Rake reception in realistic Nakagami-m channels.[2] The approach is general and enables the performance evaluation of several SS systems, not only in AWGN channels, but also in the presence of fading. Using this methodology, we investigate the performance of a UWB system in the presence of Bluetooth-like and GSM-like signals. Simulation results confirm the validity of our analytical methodology.

The paper is organized as follows. Section II presents the system model for general MF reception. The performance in an AWGN channel is derived in Section III, while the effect of multiple tone interferers in a flat-fading channel scenario is derived in Section IV. Section V addresses the analysis of a Rake receiver with a tone interferer. Some useful examples of relevant systems that can be studied are reported in Section VI. Numerical results follow in Section VII, and concluding remarks are given in Section VIII.

II. System Model

We consider a binary communication system with MF reception. To highlight the effect of NBI, we consider the reception of a wideband signal in a single-user scenario. A block diagram of the system is depicted in Fig. 1. The transmitted signal is written, in general, as

$$s(t) = \sqrt{E_\mathrm{b}}\sum_i b(t - iT_\mathrm{b}; d_i), \quad d_i \in \{0,1\} \tag{1}$$

where $b(t;d_i)$ is a unit-energy waveform used to transmit the ith information bit d_i, E_b is the energy per transmitted bit, and T_b is the bit duration. Specifically, $b(t;0)$ and $b(t;1)$ are the wide-bandwidth waveforms used to transmit bits 0 and 1, respectively. We assume that data bits d_i are independent and equiprobable.

We consider a generic linear time-invariant channel, with impulse response $h_\mathrm{D}(t)$ for the desired signal, and $h_{\mathrm{I}_n}(t)$, $n = 1,\ldots,N_\mathrm{I}$ for the interferers. We start with the case where $h_\mathrm{D}(t)$ is frequency-flat, and then extend the analysis to the case of frequency-selective multipath channels. The overall received signal $r(t)$ due to the desired signal, N_I independent interferers, and the additive noise is then given by

$$r(t) = \sqrt{E_\mathrm{b}}\sum_i r_\mathrm{b}(t - iT_\mathrm{b}; d_i) + \sum_{n=1}^{N_\mathrm{I}}\sqrt{I_n}r_{\mathrm{I}_n}(t) + \nu(t) \tag{2}$$

where $\nu(t)$ is the AWGN with two-sided power spectral density $N_0/2$, and I_n is the transmitted power of the nth interferer. Denoting the continuous convolution by $*$

$$\begin{aligned} r_\mathrm{b}(t;d_i) &= h_\mathrm{D}(t) * b(t;d_i)\\ r_{\mathrm{I}_n}(t) &= h_{\mathrm{I}_n}(t) * i_n(t) \end{aligned} \tag{3}$$

represent the response of the channels to the transmitted waveform $b(t;d_i)$ and unit-power transmitted signal $i_n(t)$, from the nth NB interferers. Note that to study both carrier-based and carrierless systems, the baseband-equivalent notation is not used. Therefore, the signals in (2) are all real-valued.

For NB interferers, it is reasonable to assume that they experience a frequency-flat fading, i.e., $h_{\mathrm{I}_n}(t) = \alpha_{\mathrm{I}_n}\delta(t-\tau_n)$, $n = 1,\ldots,N_\mathrm{I}$, where α_{I_n} are the channel gains, τ_n is the time shift, and $\delta(\cdot)$ is the Dirac delta function. Since we model the NB interferers as tones, the received signal $r_{\mathrm{I}_n}(t)$ due to the nth tone interferer becomes

$$r_{\mathrm{I}_n}(t) = \alpha_{\mathrm{I}_n}\sqrt{2}\cos(2\pi f_n(t-\tau_n)+\phi_n) \tag{4}$$

with frequency f_n and phase ϕ_n.

We model both α_{I_n} and ϕ_n as independent identically distributed (i.i.d.) random variables (r.v.s), and in particular, we consider ϕ_n to be uniformly distributed over the interval $[0, 2\pi)$. Without loss of generality, we also assume that the channel impulse response (CIR) $h_\mathrm{D}(t)$ and the interferer channel gains α_{I_n} are normalized, so that $\mathbb{E}\{\alpha_\mathrm{D}^2\} = 1$ with $\alpha_\mathrm{D}^2 = \int_{-\infty}^{\infty} h_\mathrm{D}^2(t)dt$, and $\mathbb{E}\{\alpha_{\mathrm{I}_n}^2\} = 1$, $n = 1,\ldots,N_\mathrm{I}$.[3] Therefore, in the following, we denote by E_b and I_n the average received energy per bit and the average received power for the nth interferer, respectively.

[2]Such a model has been validated recently by UWB propagation experiments [28].

[3]The notation $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

The received signal in (2) is affected by AWGN and interference. If only AWGN is present, the optimum receiver consists of a filter, matched to the difference $r_\mathrm{b}(t;0)-r_\mathrm{b}(t;1)$ or, equivalently, a correlator followed by a sampler (see Fig. 1). The template waveform $v(t)$ of the correlator is given by

$$v(t)=r_\mathrm{b}(t;0)-r_\mathrm{b}(t;1). \qquad (5)$$

Note that in the presence of multipath, this adaptive MF is realized as the well-known Rake receiver. We now consider the detection of d_0, and we assume that pulses satisfy the Nyquist criterion, or introduce, in any case, negligible intersymbol interference (ISI). Considering perfect synchronization with the desired signal, the MF output $u(t)$ at the appropriate sampling instant t_0 can be written as

$$u(t_0)=s_0+\sum_{n=1}^{N_\mathrm{I}}\alpha_{\mathrm{I}_n}\sqrt{2I_n}\,|H(f_n)|\cos\phi_n+n_0 \qquad (6)$$

where s_0 is the desired signal

$$s_0=\sqrt{E_\mathrm{b}}\int_{-\infty}^{t_0} r_\mathrm{b}(t;d_0)v(t)dt \qquad (7)$$

$H(f)$ is the transfer function of the MF, and n_0 is the noise sample with zero mean and variance $\sigma^2 = (N_0/2)\int_{-\infty}^{\infty}v^2(t)dt$. Note that the integration range is determined by the support of the template function $v(t)$. With a slight abuse of notation, the phase term $\arg\{H(f_n)\}+2\pi f_n(t_0-\tau_n)$ is absorbed by the random phase ϕ_n in (6).

The MF is matched to the received waveform, so its transfer function can be easily evaluated as

$$H(f)=\mathcal{F}\left\{\underbrace{r_\mathrm{b}(t_0-t;0)-r_\mathrm{b}(t_0-t;1)}_{=v(t_0-t)}\right\} \qquad (8)$$

where $\mathcal{F}\{\cdot\}$ denotes the Fourier transform. In particular

$$|H(f)|=|H_0(f)|\cdot|\mathcal{F}\{h_\mathrm{D}(t)\}| \qquad (9)$$

where $H_0(f) = \mathcal{F}\{b(t;0)-b(t;1)\}$. Note that (9) is composed of two factors. The first ($|H_0(f)|$) depends on the waveforms used, while the second depends on the CIR for the desired signal. We remark that $H_0(f)$ is the transfer function of the MF for AWGN and flat-fading scenarios.

One important consequence of (6) is that the performance of MF receivers depends on the product of the tone-interferer power and the MF transfer function in (9) at the frequencies of the interferers. Based on this result, in the following, we will examine the performance of specific wideband systems in different scenarios.

III. Performance in AWGN

In this section, we derive the exact expression for the BEP of an MF receiver in the presence of N_I tone interferers and AWGN. In this case, $h_\mathrm{D}(t)=\delta(t)$ and $\alpha_{\mathrm{I}_n}=1, n=1,\dots,N_\mathrm{I}$. Noting that $r_\mathrm{b}(t;d_i) = b(t;d_i)$, from (5) and (7), the desired signal can be written as

$$s_0=\sqrt{E_\mathrm{b}}(1-\rho) \qquad (10)$$

where ρ is the correlation coefficient between the two waveforms $b(t;0)$ and $b(t;1)$, i.e., $\rho \triangleq \int_{-\infty}^{\infty} b(t;0)b(t;1)dt$.[4]

Note that $\rho\in[-1,1]$, where $\rho=0$ corresponds to orthogonal signaling. The values $\rho\in(0,1]$ and $\rho\in[-1,0)$ correspond, respectively, to modulation schemes that are inferior and superior to orthogonal modulation, and finally, $\rho=-1$ corresponds to the antipodal modulation. For the performance evaluation, we assume, without loss of generality, that $d_0=0$.

Now, starting from (6), the BEP can be written as

$$P_e=\mathbb{P}\{s_0+\zeta+n_0<0|d_0=0\}=F_\varphi(-s_0) \qquad (11)$$

where $\zeta \triangleq \sum_{n=1}^{N_I}\sqrt{2I_n}|H_0(f_n)|\cos\phi_n$ and $F_\varphi(x)$ is the cumulative distribution function (CDF) of $\varphi \triangleq \zeta+n_0$.[5] Using the fact that the interferers and noise are independent, the characteristic function (CF) of φ is given by

$$\Phi_\varphi(\omega)=\mathbb{E}\{e^{j\omega\varphi}\}=\Phi_\zeta(\omega)\cdot\exp\left(-\frac{\omega^2\sigma^2}{2}\right) \qquad (12)$$

where $\Phi_\zeta(\omega)$ is the CF of the interference ζ, and $\sigma^2=N_0(1-\rho)$.

A formula for the error probability (11) can be obtained directly from the CF $\Phi_\varphi(\omega)$ using the inversion theorem [29]. By observing that φ is an even r.v., the BEP can be written as

$$P_e=\frac{1}{2}-\frac{1}{\pi}\int_0^\infty \Phi_\zeta\left(\frac{\omega}{s_0}\right)\frac{\sin\omega}{\omega}\exp\left(-\frac{\omega^2}{2}\frac{\sigma^2}{s_0^2}\right)d\omega. \qquad (13)$$

Since all the r.v.s ϕ_n are i.i.d. with uniform distribution over $[0,2\pi)$, the CF of the r.v. ζ can be evaluated in closed form as

$$\begin{aligned}\Phi_\zeta(\omega)&=\mathbb{E}\left\{\exp\left(j\omega\sum_{n=1}^{N_\mathrm{I}}\sqrt{2I_n}\,|H_0(f_n)|\cos\phi_n\right)\right\}\\&=\prod_{n=1}^{N_\mathrm{I}}J_0\left(|\omega|\sqrt{2I_n\,|H_0(f_n)|^2}\right)\end{aligned} \qquad (14)$$

where $J_0(\omega)$ is the zeroth-order Bessel function of the first kind. Substituting (14) into (13), the exact BEP expression can be written as a function of the signal-to-noise ratio (SNR) E_b/N_0, and the signal-to-interference ratios (SIRs) C/I_n, as

$$\begin{aligned}P_e=\frac{1}{2}-\frac{1}{\pi}\int_0^\infty\prod_{n=1}^{N_\mathrm{I}}J_0\left(\omega\sqrt{\frac{I_n}{C}\frac{|H_0(f_n)|^2}{T_\mathrm{b}}\frac{2}{(1-\rho)^2}}\right)\\ \times\frac{\sin\omega}{\omega}\exp\left(-\frac{\omega^2}{2}\cdot\frac{N_0}{E_\mathrm{b}}\frac{1}{1-\rho}\right)d\omega\end{aligned} \qquad (15)$$

where $C=E_\mathrm{b}/T_\mathrm{b}$ denotes the useful received power. Note that in the absence of interference, we have $\zeta=0$, and then $\Phi_\zeta(\omega)=1$, therefore, from (11) with $\zeta=0$, the BEP can be evaluated easily as $P_e=\mathcal{Q}(\sqrt{s_0^2/\sigma^2}$, where $\mathcal{Q}(\cdot)$ is the well-

[4]Distortions, e.g., caused by antennas in UWB systems can be taken into account by considering $b(t;d_i)$ as the received waveform. This will be illustrated clearly in Section VI.

[5]The notation $\mathbb{P}\{X|Y\}$ denotes the probability of X given Y.

known Gaussian Q-function. Furthermore, (13) with $\Phi_\zeta(\omega) = 1$ implies that

$$\mathcal{Q}\left(\sqrt{\frac{s_0^2}{\sigma^2}}\right) + \frac{1}{\pi}\int_0^\infty \frac{\sin\omega}{\omega}\exp\left(-\frac{\omega^2}{2}\frac{\sigma^2}{s_0^2}\right)d\omega = \frac{1}{2}. \quad (16)$$

Therefore, (15) can be rewritten as

$$\begin{aligned} P_e = &\mathcal{Q}\left(\sqrt{\frac{E_\mathrm{b}}{N_0}(1-\rho)}\right) \\ &+ \frac{1}{\pi}\int_0^\infty \left[1 - \prod_{n=1}^{N_\mathrm{I}} J_0\left(\omega\sqrt{\frac{I_n}{C}\frac{|H_0(f_n)|^2}{T_\mathrm{b}}\frac{2}{(1-\rho)^2}}\right)\right] \\ &\times \frac{\sin\omega}{\omega}\exp\left(-\frac{\omega^2}{2}\cdot\frac{N_0}{E_\mathrm{b}}\frac{1}{1-\rho}\right)d\omega. \end{aligned} \quad (17)$$

This formula gives the exact BEP expression for a coherent detection of binary signals in the presence of N_I tone interferers, with arbitrary amplitudes and frequencies, and AWGN. Moreover, the right side of (17) is composed of two terms: the first is the BEP for the binary system in AWGN only, and the second is the increase in BEP due to the presence of N_I interferers. Note also that (17) is numerically more stable than (15) for small values of P_e. In fact, in (15), the BEP is evaluated as a difference of values that are close to 0.5, hence, it is more sensitive to errors due to numerical evaluation of the integral.

IV. Performance in Flat-Fading Channels

In this section, we evaluate the performance of MF systems in the presence of independent Rayleigh fading on the multiple interferers and AWGN. We first consider the scenario in which the desired signal is unfaded, and then follow up with the scenario in which the desired signal undergoes Nakagami-m fading. These scenarios describe a common situation encountered in practice, when a wideband system, such as a wireless personal area network (WPAN), operates in an indoor environment.[6] In both scenarios, NB interferers originate from other sources such as mobile phones, which could be affected by Rayleigh fading.

In this case, $h_\mathrm{D}(t) = \alpha_\mathrm{D}\delta(t)$ and therefore, starting from (2), the received signal can be expressed as

$$\begin{aligned} r(t) = &\alpha_\mathrm{D}\sqrt{E_\mathrm{b}}\sum_i b(t - iT_\mathrm{b}; d_i) \\ &+ \sum_{n=1}^{N_\mathrm{I}} \alpha_{\mathrm{I}_n}\sqrt{2I_n}\cos(2\pi f_n(t-\tau_n) + \phi_n) + \nu(t) \end{aligned} \quad (18)$$

where E_b is the average received energy per bit, α_{I_n} are independent Rayleigh distributed r.v.s with unit power, and I_n is the average received power of the nth interferer.

A. Unfaded/Faded Scenario

Let us first consider the scenario in which the amplitude of the desired signal remains constant ($\alpha_\mathrm{D} = 1$) and the interferers have Rayleigh-distributed amplitudes. This represents the case of short-range transmissions, often characterized by line-of-sight propagation with fixed positions for both the transmitter and receiver.

Analyzing the MF output $u(t)$ at the appropriate sampling instant t_0, each term $\alpha_{\mathrm{I}_n}\cos\phi_n$ is a zero-mean Gaussian r.v. with variance 1/2. Therefore, the interferer term $\zeta \triangleq \sum_{n=1}^{N_\mathrm{I}} \alpha_{\mathrm{I}_n}\sqrt{2I_n}|H_0(f_n)|\cos\phi_n$ at the output of the MF is zero-mean Gaussian with power $\sigma_\zeta^2 = \sum_{n=1}^{N_\mathrm{I}} I_n|H_0(f_n)|^2$, implying that the total disturbance $\zeta + n_0$ is Gaussian-distributed with power $\sigma_\zeta^2 + \sigma^2$. Since the total disturbance is distributed as Gaussian, the BEP can be written as $P_e = \mathcal{Q}(\sqrt{2\eta})$, where

$$\eta = \left(\frac{N_0}{E_\mathrm{b}}\frac{2}{1-\rho} + \frac{2}{(1-\rho)^2}\sum_{n=1}^{N_\mathrm{I}}\frac{I_n}{C}\frac{|H_0(f_n)|^2}{T_\mathrm{b}}\right)^{-1} \quad (19)$$

is the average signal-to-interference-plus-noise ratio (SINR) as a function of the SNR E_b/N_0 and the average SIRs C/I_n corresponding to the nth interferer. This formula gives the exact BEP expression for the coherent receiver in the presence of N_I Rayleigh-faded tone interferers, with arbitrary amplitudes and frequencies, and AWGN.

B. Faded/Faded Scenario

This represents a scenario in which the desired signal and interferers are all affected by fading, for example, due to nonline-of-sight propagation. It is well known that Rake receivers are typically used for wideband transmission systems in a multipath fading channel, to exploit multipath diversity. The detailed analysis of this scenario is carried out in Section V. Here, we would like to point out that a single correlator receiver, although not optimum, can be a potential solution for low-cost transmission systems. This is the case of wireless sensor networks, where each node has energy and space constraints that necessitate the use of simple devices. In this scenario, a single correlator receiver is able to capture energy only from one of the many resolvable paths. In accordance with the channel model presented in [28], which suggests the use of the Nakagami-m distribution for path amplitudes, we derive the BEP for a correlator receiver in a multipath fading scenario.

We assume that the desired user's fading amplitude α_D follows the Nakagami-m distribution with probability density function given by [30]

$$f_{\alpha_\mathrm{D}}(r) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m r^{2m-1}\exp\left(-\frac{mr^2}{\Omega}\right), \quad r \geq 0 \quad (20)$$

where $\Omega = \mathbb{E}\{\alpha_\mathrm{D}^2\} = 1$ and $m \geq 0.5$ is the fading parameter that controls the severity of the fading conditions. In the worst case, $m = 0.5$, the fading distribution follows a one-sided Gaussian distribution and $m = 1$ corresponds to Rayleigh fading, while the best case, when m approaches infinity, corresponds to an unfaded channel.

Starting with (19), the BEP conditioned on the instantaneous normalized fading can be written as $P_{e|\alpha_\mathrm{D}} = \mathcal{Q}(\alpha_\mathrm{D}\sqrt{2\eta})$, where η has the same expression (19). By averaging $P_{e|\alpha_\mathrm{D}}$ with

[6]The former scenario represents short-range communication with fixed line-of-sight propagation, whereas the latter corresponds to communication between slowly moving terminal with nonline-of-sight propagation.

respect to the r.v. α_D, i.e., $P_e = \mathbb{E}_{\alpha_D}\{P_{e|\alpha_D}\}$, we obtain the average BEP as [31]

$$P_e = \frac{\Gamma\left(m+\frac{1}{2}\right)}{2\sqrt{\pi}\Gamma(m+1)}\left(\frac{m}{\eta}\right)^m {}_2F_1\left(m, m+\frac{1}{2}; m+1; -\frac{m}{\eta}\right) \quad (21)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [32, eq. 9.14]. Note that for integer values of m, (21) can be simplified as

$$P_e = \left[\frac{1}{2} - \frac{1}{2}\left(1+\frac{m}{\eta}\right)^{-\frac{1}{2}}\right]^m \times \sum_{j=0}^{m-1}\binom{m-1+j}{j}\left[\frac{1}{2}+\frac{1}{2}\left(1+\frac{m}{\eta}\right)^{-\frac{1}{2}}\right]^j. \quad (22)$$

V. Performance in Frequency-Selective Fading Channels

In this section, we derive a closed-form BEP expression for a Rake receiver in the presence of a tone interferer and AWGN. Our approach is based on perturbation theory, and is valid for a broad range of signaling schemes such as DS and TH, which have been under consideration for UWB systems.

We consider a frequency-selective multipath fading channel for the desired signal with impulse response

$$h_D(t) = \sum_{k=1}^{L} h_k \delta(t - t_k) \quad (23)$$

where $h_k = \alpha_k \cdot e^{j\theta_k}$ for $k = 1, 2, \ldots, L$ are statistically independent r.v.s describing the L path gains, and t_k is the delay of the kth path. In particular, the parameter α_k represents the fading amplitude, while $\theta_k \in \{0, \pi\}$ with equal probability accounts for the random phase inversion that can occur due to reflections.

The received waveform can be written using (3) and (23) as[7]

$$r_b(t; d_0) = \sum_{k=1}^{L} h_k b(t - t_k; d_0). \quad (24)$$

The template waveform corresponding to an ideal Rake receiver (i.e., the waveform to which the Rake is matched) can then be written as

$$v(t) = \sum_{k=1}^{L} h_k \left[b(t-t_k; 0) - b(t-t_k; 1)\right]. \quad (25)$$

According to (9), (25) implies that the Rake receiver can be represented in the frequency domain as a filter with transfer function

$$|H(f)| = |H_0(f)| \cdot |H_D(f, \mathbf{h}, \mathbf{t})| \quad (26)$$

[7]We assume that distortions, if present, are the same for all waveforms received through the multiple paths.

where

$$H_D(f, \mathbf{h}, \mathbf{t}) = \mathcal{F}\{h_D(t)\} = \sum_{k=1}^{L} h_k e^{-j2\pi f t_k} \quad (27)$$

with random vectors (R.V.s) $\mathbf{h} = (h_1, h_2, \ldots, h_L)$ and $\mathbf{t} = (t_1, t_2, \ldots, t_L)$ denoting instantaneous path gains and delays, respectively.

Substituting (24) and (25) into (7), the desired signal s_0 can be written as

$$s_0 = \sqrt{E_b}(1-\rho)\alpha_D^2 \quad (28)$$

where $\alpha_D^2(\mathbf{h}) = \sum_{k=1}^{L} h_k^2$ and ρ is the correlation coefficient.[8] The contribution from the tone interferer is $\alpha_I\sqrt{2I}|H(f_I)|\cos\phi$, where α_I is the Rayleigh-distributed amplitude, I is the average power, f_I is the frequency, and ϕ is the random phase uniformly distributed over $[0, 2\pi)$. The noise power at the output of the Rake combiner can be evaluated as

$$\sigma^2 = N_0(1-\rho)\alpha_D^2. \quad (29)$$

Note from (26) that $|H(f_I)|$ is a r.v. which depends on the instantaneous CIR $h_D(t)$ through $\mathbf{h}$ and $\mathbf{t}$. When conditioned on the r.v. $|H(f_I)|$, the interferer term $\alpha_I\sqrt{2I}|H(f_I)|\cos\phi$ is conditionally Gaussian with variance $I|H(f_I)|^2$. Therefore, the total disturbance due to interference plus noise is also conditionally Gaussian with variance $I|H_0(f_I)|^2 \cdot |H_D(f_I, \mathbf{h}, \mathbf{t})|^2 + N_0(1-\rho)\alpha_D^2$. The conditional BEP, conditioned on the instantaneous CIR, can then be written as $P_{e|\mathbf{h},\mathbf{t}} = \mathcal{Q}(\sqrt{2\tilde{\eta}(\mathbf{h},\mathbf{t})})$, where

$$\tilde{\eta}(\mathbf{h}, \mathbf{t}) = \frac{\alpha_D^2(\mathbf{h})}{\frac{N_0}{E_b}\frac{2}{1-\rho} + \frac{I}{C}\frac{2|H_0(f_I)|^2}{T_b(1-\rho)^2}\frac{|H_D(f_I,\mathbf{h},\mathbf{t})|^2}{\alpha_D^2(\mathbf{h})}}. \quad (30)$$

The next step is to perform expectations of $P_{e|\mathbf{h},\mathbf{t}}$ over R.V.s $\mathbf{h}$ and $\mathbf{t}$, in order to obtain the average performance over all the possible CIRs $h_D(t)$.

The contribution of the interference in (30) depends on the instantaneous CIR of the desired signal, making it cumbersome and difficult to evaluate the expectation $\mathbb{E}_{\mathbf{h},\mathbf{t}}\{P_{e|\mathbf{h},\mathbf{t}}\}$ in closed form. However, we can proceed with two steps: first we perform the average over the time delays $\mathbf{t}$ for fixed path gains $\mathbf{h}$, and then we average over $\mathbf{h}$, i.e., $P_e = \mathbb{E}_{\mathbf{h}}\{\mathbb{E}_{\mathbf{t}}\{P_{e|\mathbf{h},\mathbf{t}}\}\}$. In this case, the inner expectation involves only the R.V. $\mathbf{t}$ through the function $|H_D(f_I, \mathbf{h}, \mathbf{t})|^2$; therefore, without loss of generality, in the following, we define a r.v. $\xi = |H_D(f_I, \mathbf{h}, \mathbf{t})|^2$ that depends on R.V. $\mathbf{t}$ with fixed, but arbitrary, $\mathbf{h}$. Hence, $P_{e|\mathbf{h}} = \mathbb{E}_\xi\{\mathcal{P}(\xi)\}$, where $\mathcal{P}(\xi) \triangleq \mathcal{Q}(\sqrt{2\check{\eta}(\xi)})$ with

$$\check{\eta}(\xi) = \frac{\alpha_D^2}{\frac{N_0}{E_b}\frac{2}{1-\rho} + \frac{I}{C}\frac{2|H_0(f_I)|^2}{T_b(1-\rho)^2}\frac{\xi}{\alpha_D^2}}. \quad (31)$$

The expectation of (31) over the r.v. ξ can be conveniently approximated by means of perturbation theory [33], [34] without requiring integration. In fact, by expanding $\mathcal{P}(\xi)$ about μ_1 in

[8]In deriving (28), we made the standard assumption that the transmitted waveforms possess ideal correlation properties, and that the Rake paths are resolvable. This implies that ISI, as well as self-interference, are negligible.

terms of central differences up to the third order and taking the expectation, we obtain

$$P_{e|\mathbf{h}} = \mathbb{E}_{\mathbf{t}}\{P_{e|\mathbf{h},\mathbf{t}}\} \simeq \mathcal{P}(\mu_1) + \frac{\mu_2}{2}\cdot\frac{\mathcal{P}(\mu_1+z)-2\mathcal{P}(\mu_1)+\mathcal{P}(\mu_1-z)}{z^2} + \frac{\mu_3}{12}\cdot\frac{\mathcal{P}(\mu_1+2z)-2\mathcal{P}(\mu_1+z)+2\mathcal{P}(\mu_1-z)-\mathcal{P}(\mu_1-2z)}{z^3} \quad (32)$$

where

$$\mu_1(\mathbf{h}) = \mathbb{E}_{\mathbf{t}}\left\{|H_{\mathrm{D}}(f_{\mathrm{I}},\mathbf{h},\mathbf{t})|^2\right\}$$
$$\mu_2(\mathbf{h}) = \mathbb{E}_{\mathbf{t}}\left\{\left[|H_{\mathrm{D}}(f_{\mathrm{I}},\mathbf{h},\mathbf{t})|^2 - \mu_1(\mathbf{h})\right]^2\right\}$$
$$\mu_3(\mathbf{h}) = \mathbb{E}_{\mathbf{t}}\left\{\left[|H_{\mathrm{D}}(f_{\mathrm{I}},\mathbf{h},\mathbf{t})|^2 - \mu_1(\mathbf{h})\right]^3\right\} \quad (33)$$

are the first-, second-, and third-order central moments, respectively, of the r.v. $|H_{\mathrm{D}}(f_{\mathrm{I}},\mathbf{h},\mathbf{t})|^2$. In the Appendix, the central moments in (33) are evaluated using approximations that enable further averaging over the path amplitudes $\mathbf{h}$. In general, the constant z is an arbitrary parameter, and $z=\sqrt{3\mu_2}$ is chosen in [33] to minimize the error due to truncation of the expansion.

One shortcoming of the expansion (32) is that $z=\sqrt{3\mu_2}$ can be greater than μ_1 under some conditions. This results in an imaginary argument of the Q-function that cannot be evaluated [35]. However, the terms $\mathcal{P}(\mu_1-z)$ and $\mathcal{P}(\mu_1-2z)$ can be replaced by $\mathcal{P}(0)$ to avoid numerical problems without affecting the accuracy of the results [36, eq. (23)]. Using the above analysis in conjunction with the approximations discussed in the Appendix, (32) can be written in the form

$$P_{e|\mathbf{h}} \simeq \sum_{i=1}^{N} p_i \mathcal{P}\left(q_i \alpha_{\mathrm{D}}^2\right) \quad (34)$$

where p_i and q_i are weights, while N is the number of terms in the expansion (32). For the third-order expansion considered, we have $N=4$ terms, with the weights $\mathbf{p}=(1/6+b, 2/3, 1/6-2b, b)$ and $\mathbf{q}=(0,1,1+\sqrt{3a},1+2\sqrt{3a})$. The parameters $a=1-\beta$ and $b=(1-3\beta+2\kappa)/(18\sqrt{3}(1-\beta)^{3/2})$ are functions of β and κ that depend only on the normalized power dispersion profile (PDP) of the channel, i.e., $\beta=\sum_{k=1}^{L}\Omega_k^2$ and $\kappa=\sum_{k=1}^{L}\Omega_k^3$. For an exponential PDP with path gains

$$\Omega_k = \frac{e^{\frac{1}{\epsilon}}-1}{1-e^{-\frac{L}{\epsilon}}}e^{-\frac{k}{\epsilon}}, \quad k=1,\ldots,L \quad (35)$$

where ϵ is a decay constant which controls the multipath dispersion [15], [28], the constants β and κ can be evaluated as functions of L and ϵ, i.e.,

$$\beta(L,\epsilon) = \frac{\tanh\left(\frac{1}{2\epsilon}\right)}{\tanh\left(\frac{L}{2\epsilon}\right)}$$
$$\kappa(L,\epsilon) = \frac{1+2\cosh\left(\frac{L}{\epsilon}\right)}{1+2\cosh\left(\frac{1}{\epsilon}\right)}\cdot\frac{\sinh^2\left(\frac{1}{2\epsilon}\right)}{\sinh^2\left(\frac{L}{2\epsilon}\right)}. \quad (36)$$

Note that the expression in (34) is a function of $\mathbf{h}$ through the term α_{D}^2, and hence, the outer expectation $P_e=\mathbb{E}_{\mathbf{h}}\{P_{e|\mathbf{h}}\}$ can be written as the expectation of (34) with respect to α_{D}^2. The distribution of α_{D}^2 depends on the type of channel. For UWB channels, it has been shown recently that the amplitude distribution of the resolved multipaths can be modeled by the Nakagami-m distribution [28]. Accordingly, we consider independent Nakagami-distributed paths, α_k, with average power $\Omega_k=\mathbb{E}\{h_k^2\}=\mathbb{E}\{\alpha_k^2\}$ and Nakagami parameter m_k. Using the alternative expression for the Gaussian Q-function [37], one can derive the expectation of (34) over $\mathbf{h}$ as

$$P_e \simeq \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\sum_{i=1}^{N}p_i\prod_{k=1}^{L}\left(\frac{\eta(q_i)}{\sin^2\theta}\cdot\frac{\Omega_k}{m_k}+1\right)^{-m_k}d\theta \quad (37)$$

where

$$\eta(x) = \left(\frac{N_0}{E_{\mathrm{b}}}\frac{2}{1-\rho}+\frac{I}{C}\frac{|H_0(f_{\mathrm{I}})|^2}{T_{\mathrm{b}}}\frac{2x}{(1-\rho)^2}\right)^{-1} \quad (38)$$

is the mean SINR as a function of the average SNR E_{b}/N_0 and the average SIR C/I.

Note that the approximation developed above is more accurate than the standard Gaussian approximation for the interferer, which would be equivalent to considering only the first term of the expansion in (32). Furthermore, for $L=1$, we have $\xi=\alpha_{\mathrm{D}}^2$ and $P_{e|\mathbf{h},\mathbf{t}}$ can be averaged over $\mathbf{h}$ in closed form; therefore, (37) with $N=1$, $p_1=1$, and $q_1=1$ gives an alternative exact BEP expression for the flat-fading scenario derived in Section IV-B with $N_{\mathrm{I}}=1$.

VI. Examples

Using the general approach developed in previous sections, we can now evaluate the BEP for some important SS systems, such as TH-pulse position moduation (PPM), TH-pulse amplitude modulation (PAM), and DS-binary phase-shift keying (BPSK).

A. TH With PPM and PAM

Here, we consider binary TH-PPM and TH-PAM systems which are the most popular UWB transmission schemes [2], [21]. Using the bit waveforms $b(t;d_i)\triangleq b(t-d_i\delta)$ for PPM and $b(t;d_i)\triangleq d_i b(t)$ for PAM, the transmitted TH signal can be written as

$$s(t)=\sqrt{E_{\mathrm{b}}}\sum_i b(t-iN_{\mathrm{s}}T_{\mathrm{f}}-d_i\delta) \quad \text{(TH-PPM)}$$
$$s(t)=\sqrt{E_{\mathrm{b}}}\sum_i d_i b(t-iN_{\mathrm{s}}T_{\mathrm{f}}) \quad \text{(TH-PAM)} \quad (39)$$

with the unit-energy waveform for each bit given by

$$b(t)=\sum_{j=0}^{N_{\mathrm{s}}-1}w(t-jT_{\mathrm{f}}-c_jT_{\mathrm{c}}) \quad (40)$$

where N_{s} is the number of pulses required to transmit a single information bit d_i, belonging to the set $\{0,1\}$ for TH-PPM or the set $\{-1,1\}$ (through a simple mapping) for TH-PAM. The parameter δ is the pulse-position offset, $w(t)$ is the signal pulse with energy $1/N_s$, and E_b is the received bit energy. The pulse-repetition time (frame length) T_f and the bit duration T_b are related by $T_b = N_s T_f$. Finally, $\{c_j\}$ is the TH sequence, and T_c is the TH chip width.

The transfer function $H_0(f)$ in (9) for both cases can be easily derived as

$$\begin{aligned} |H_0(f)| &= 2\,|W(f)|\,|\sin(\pi f \delta)| \\ &\quad \times \left| \sum_{k=0}^{N_s-1} e^{j2\pi f(kT_f + c_k T_c)} \right| \quad \text{(TH-PPM)} \\ |H_0(f)| &= 2\,|W(f)| \\ &\quad \times \left| \sum_{k=0}^{N_s-1} e^{j2\pi f(kT_f + c_k T_c)} \right| \quad \text{(TH-PAM)} \end{aligned} \tag{41}$$

where $W(f)$ is the Fourier transform of the pulse $w(t)$.

The received pulse $w(t)$ can be modeled as the second derivative of a Gaussian monocycle with energy $1/N_s$ as [1]

$$w(t) = \sqrt{\frac{1}{N_s E_w}} \left[1 - 4\pi \left(\frac{t}{\tau_w} \right)^2 \right] e^{-2\pi \left(\frac{t}{\tau_w} \right)^2} \tag{42}$$

where $E_w = 3\tau_w/8$ and τ_w is related to the pulse duration. The Fourier transform of $w(t)$ is

$$W(f) = \sqrt{\frac{1}{N_s E_w}} \frac{\pi}{\sqrt{2}} \tau_w^3 f^2 e^{-\frac{\pi}{2} f^2 \tau_w^2}. \tag{43}$$

Therefore, the BEP for TH-PPM and TH-PAM in the various scenarios considered in previous sections are given by (17), (19), (21), and (37), together with (41). Recall from (6) that the effect of the ith interferer depends on its frequency, and thus on the system parameters through $|H_0(f)|$ in (41).

B. DS With BPSK

The transmitted DS-BPSK signal is given by

$$s(t) = \sqrt{E_b} \sum_i d_i b(t - iT_b) \tag{44}$$

where E_b is the bit energy, $d_i \in \{-1, +1\}$, and T_b is the bit duration. The unit-energy bit waveform can be written as

$$b(t) = \sqrt{\frac{2}{M E_g}} \sum_{j=0}^{M-1} c_j g(t - jT_c) \cos(2\pi f_0 t + \varphi) \tag{45}$$

where f_0 is the carrier frequency, φ is the arbitrary phase of the carrier, and T_c is the chip duration. The pulse $g(t)$ represents the chip waveform with energy E_g, while $\{c_j\}_{j=0}^{M-1}$ is the desired user's spreading sequence of length M with $c_j \in \{-1, +1\}$. For the sake of simplicity, we consider $T_b = MT_c$, i.e., the entire spreading sequence is transmitted during a bit.

The template waveform $v(t)$, matched to the sequence of M chips, is given by $v(t) = b(t;0) - b(t;1) = 2b(t)$. Therefore, the transfer function is

$$|H_0(f)| = \sqrt{\frac{2}{M E_g}} \left| S(f - f_0) e^{-j\varphi} + S(f + f_0) e^{j\varphi} \right| \tag{46}$$

where $S(f)$ is the Fourier transform of the spreading waveform $\sum_{j=0}^{M-1} c_j g(t - jT_c)$, i.e.,

$$S(f) = G(f) \cdot \sum_{k=0}^{M-1} c_k e^{-j2\pi f k T_c} \tag{47}$$

with $G(f)$ denoting the Fourier transform of the chip waveform $g(t)$.

Now, the term $S(f + f_0)$ can be neglected, since it represents a high-frequency term, and we obtain

$$|H_0(f)| = \sqrt{\frac{2}{M E_g}} \, |G(f - f_0)| \left| \sum_{k=0}^{M-1} c_k e^{-j2\pi (f - f_0) k T_c} \right|. \tag{48}$$

For example, for a unit-amplitude rectangular pulse with energy $E_g = T_c |G(f)| = T_c$, $|\text{sinc}(fT_c)|$, and the performance in the scenarios considered are given by (17), (19), (21), and (37), together with (48). In particular, note that (17) with (48) gives an exact result, whereas prior results in [18] and [19] give only an upper bound.

As a special case of the DS-BPSK analyzed above with $M = 1$, we are able to evaluate the performances of a BPSK system with transmitted signal $s(t) = \sqrt{E_b} \sum_i d_i b(t - iT_b)$, and the unit-energy bit waveform expressed as

$$b(t) = \sqrt{\frac{2}{E_g}} g(t) \cos(2\pi f_0 t + \varphi) \tag{49}$$

where $g(t)$ is the bit waveform, and f_0 and φ are the frequency and phase of the carrier. In this case, the transfer function $H_0(f)$ becomes

$$|H_0(f)| = \sqrt{\frac{2}{E_g}} \, |G(f - f_0)| \,. \tag{50}$$

Equation (17) together with (50) gives the exact BEP expression for BPSK in the presence of multiple tone interferers with arbitrary frequencies and AWGN. The special case of this, where there are multiple cochannel interferers having the same frequency as the desired carrier frequency, can be found in [38].

For SS systems, note that BEP expressions (17), (19), (21), and (37) depend on the sequence (for TH or DS) $\{c_j\}$ of the desired user through $H_0(f)$. Therefore, in a multiple-access system, different users experience different BEP degradation with respect to the same tone interferer. Average performance must be obtained by averaging over the sequences of all the users. From the system design point of view, this suggests the construction of sequences that reduce the effect of NBI, introducing, for example, notch frequencies where the interferers operate [1], [23], [24].

Furthermore, looking at the transfer function of the MF, it is interesting to compare the performance of TH and DS schemes affected by NBI [20], [39]. For example, considering the transfer function of a TH system, there are frequencies at which the N_s terms in the summation of (41) are all equal to one, so that they add coherently, yielding the worst BEP. In fact, in this case, (41) with (43) imply that for a fixed T_b, the power of the interferer at the output of the MF is proportional to the length of the sequence

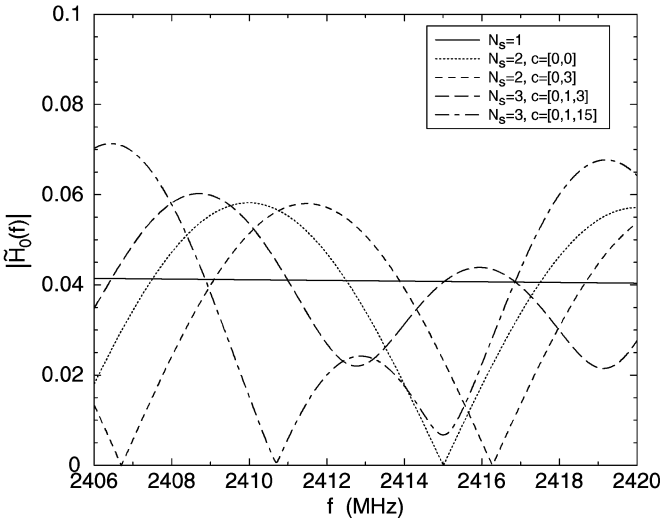

Fig. 2. Normalized transfer function of the MF around $f_{\mathrm{I}} = 2.412$ GHz for different values of N_{s} and different TH sequences.

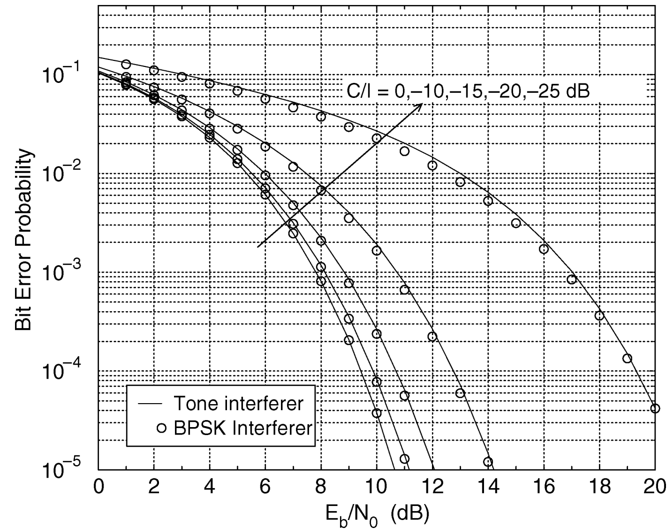

Fig. 3. BEP for the TH-PPM system considered in the unfaded/unfaded scenario with a single tone interferer. Analytical results are also compared with the simulation for a 1 Mb/s BPSK interferer.

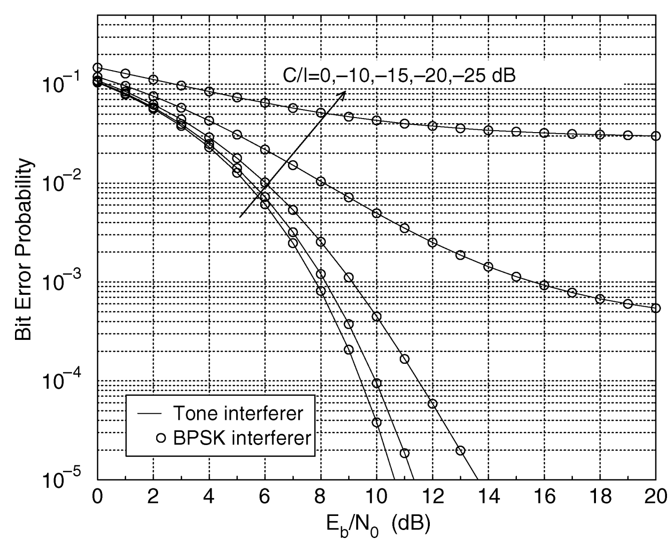

Fig. 4. BEP for the TH-PPM system considered in the unfaded/faded scenario with a single tone interferer. Analytical results are also compared with the simulation for a 1 Mb/s BPSK interferer.

N_{s}; therefore, BEP performance degrades as N_{s} increases. In contrast, the presence of the spreading code in (46) avoids this problem in a DS system, so the effect of an NB interferer in the worst case is independent of the length M.

VII. Results

In this section, we evaluate the performance of a TH-PPM system using the analytical approach developed in previous sections. We consider a second derivative Gaussian received pulse in (42) with $\tau_w = 0.5$ ns, a frame length $T_{\mathrm{f}} = 100$ ns, a pulse-position offset $\delta = 0.3$ ns, and $N_{\mathrm{s}} = 2$ pulses per bit. The resulting correlation parameter ρ is -0.587. Furthermore, we consider a user with a TH sequence of all zeros, i.e., $c_0 = c_1 = 0$ and $T_{\mathrm{c}} = 1.5$ ns. The tone interferer has frequency $f_{\mathrm{I}} = 2.412$ GHz.

To better understand the behavior of the MF as a function of N_{s} and the TH sequence of the desired user, Fig. 2 shows the normalized transfer function $\widetilde{H}_0(f) = H_0(f)/\sqrt{T_{\mathrm{b}}}$ around f_{I} using (41). For the TH-PPM system considered, the transfer function (41) is composed of two terms, the first related to the spectrum of the pulse which has a large bandwidth, while the second $|\sum_{k=0}^{N_{\mathrm{s}}-1} e^{j2\pi f(kT_{\mathrm{f}}+c_kT_{\mathrm{c}})}|$ has an oscillatory behavior which results in a frequency selectivity, as depicted in Fig. 2. This selectivity is roughly $\Delta f = ((N_{\mathrm{s}}-1)T_{\mathrm{f}})^{-1}$, which approaches infinity for $N_s = 1$ and approximately $\Delta f \approx (N_{\mathrm{s}}T_{\mathrm{f}})^{-1} = R_{\mathrm{b}}$ for $N_{\mathrm{s}} > 1$. Therefore, a simple rule of thumb to assess the validity of the tone approximation consists of verifying that the bandwidth of the interferer is less than the bit rate of the desired signal.

In the following, we evaluate the performance of TH-PPM scheme in the presence of a NB interferer and AWGN. In particular, we consider a NB interferer with a bit rate of 1 Mb/s, BPSK modulation, rectangular pulse shape, and carrier frequency f_{I}. Fig. 3 shows the simulated BEP as a function of E_{b}/N_0 for different values of the SIR C/I. Also shown in the figure are BEP results using the analytical expression in (17) based on tone interference. Note that the theoretical performance for the tone interferer and simulated performance for the 1 Mb/s BPSK interferer are within 0.25 dB at $P_e = 10^{-3}$.

We next consider the case in which the interferer goes through fading while the desired signal is unfaded. Fig. 4 shows the BEP evaluated using (19) and the simulated BEP in the presence of a NB interferer and AWGN. It can be seen by comparing Figs. 3 and 4 that the presence of fading on the interferer severely degrades performance. This is expected, since fading causes power fluctuations of the interferer, in which case, the instantaneous power could be much greater than the mean power.

We now consider the situation where both desired and interfering signals undergo fading. The BEP evaluated using (21) is reported in Fig. 5 for different values of m. As expected, comparing the BEP with previous scenarios, the performance is severely degraded by the presence of fading on the desired signal.

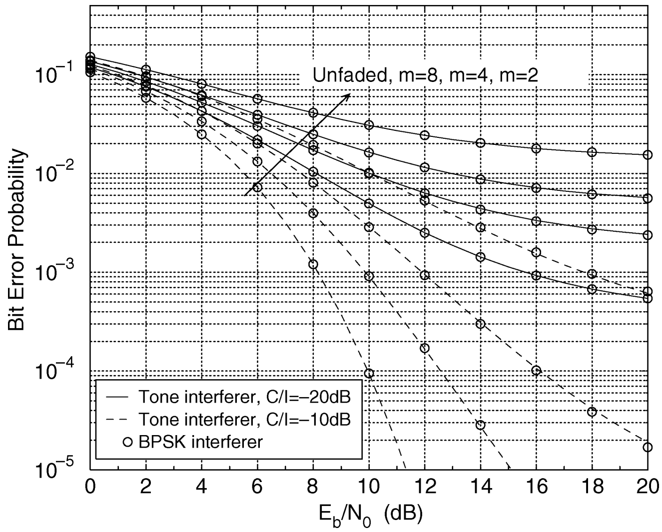

Fig. 5. BEP for the TH-PPM system with a single tone interferer in the faded/faded scenario, for different values of m and for the unfaded/faded case, i.e., $m = \infty$. Analytical results are compared with simulations for a 1 Mb/s BPSK interferer.

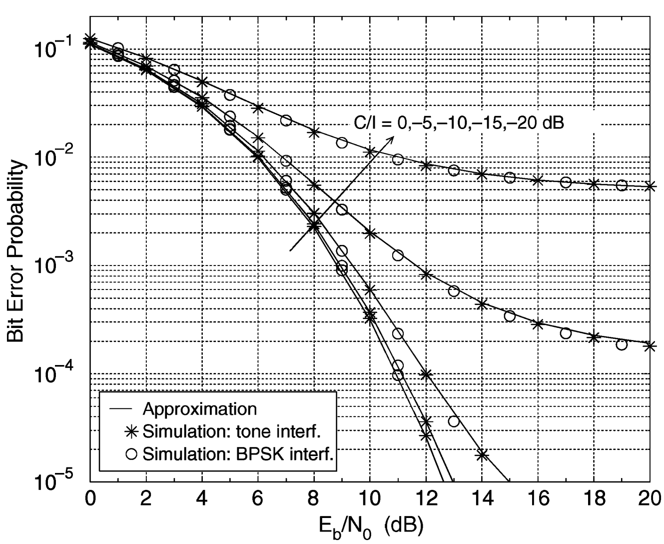

Fig. 6. BEP for the TH-PPM system considered with Rake reception and $L = 8$ paths. Analytical results are also compared with the simulation for a 1 Mb/s BPSK interferer.

Finally, the performance with Rake reception in a multipath fading scenario is considered. In particular, we consider a multipath channel with an exponential PDP (35) with L independent Nakagami distributed paths having random delays and Nakagami parameters

$$m_k = m_1 e^{-\frac{(k-1)}{\gamma}}, \quad k = 1, \ldots, L \tag{51}$$

where γ controls the decaying of the m-parameters. To validate our methodology based on perturbation theory, the analytical result of (37) is compared with signal-level simulations. In particular, Fig. 6 shows the BEP for the system with $L = 8$ paths, $\epsilon = 3$, $\gamma = 4$, and $m_1 = 3$, while Fig. 7 shows the same performance in a more dispersive channel with $L = 32$ paths, $\epsilon = 10$,

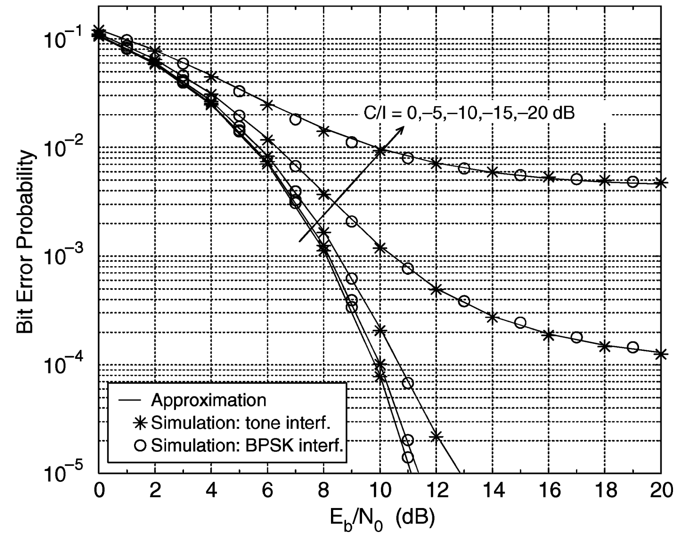

Fig. 7. BEP for the TH-PPM system considered with Rake reception and $L = 32$ paths. Analytical results are also compared with the simulation for a 1 Mb/s BPSK interferer.

$\gamma = 18$, and $m_1 = 3$. Despite the high number of paths (Rake fingers), our analytical result (37) agrees with the simulation results based on a single tone interferer (stars) for a wide range of SIRs, from −20 to 0 dB. Also for this scenario, the comparison of our closed-form result (37) with simulation for the NB interferer shows that the assumption of tone interferers is a good approximation for NB interferers.

VIII. Conclusions

In this paper, we derived closed-form expressions for the BEP of a general binary coherent system, based on MF reception with tone interferers in different scenarios. In the frequency-selective multipath fading scenario, Rake reception was considered, and an approximation based on perturbation theory was derived. The approach developed is general, and leads to performance evaluation of important SS systems, such as DS and TH. The analysis with the NB interferer suggests that, for example, wireless systems such as Bluetooth (a standard for WPANs) or GSM (a second-generation cellular system) with bandwidths of 1 MHz and 200 kHz, respectively, can be accurately represented by tone approximation. It is shown that our analytical results are useful for the evaluation of a possible coexistence between wideband systems and existing NB wireless systems.

Appendix

Let us start by considering the r.v. $\xi = |\sum_{k=1}^{L} \alpha_k e^{-j\Theta_k}|^2$, where $\Theta_k = 2\pi f_{\rm I} t_k + \theta_k$ and $\theta_k \in \{0, \pi\}$ with equal probability. In general, the delays t_k can be fixed or random with $t_k \in [0, T_{\rm d})$, where $T_{\rm d}$ is a maximum excess delay. For a typical tone interferer with frequency $f_{\rm I}$ higher than 100 MHz and $T_{\rm d}$ for a typical indoor channel on the order of 200–300 ns, the product $f_{\rm I} t_k$ is much greater than unity. This suggests the assumption of considering Θ_k as i.i.d. r.v.s uniformly distributed over $[0, 2\pi)$.

Given the r.v. $\xi = \sum_{k=1}^{L} \sum_{n=1}^{L} \alpha_k \alpha_n e^{-j(\Theta_k - \Theta_n)}$ and the statistics of the random phases Θ_k, the moments μ_1, μ_2, and μ_3 can be evaluated for a fixed $\mathbf{h}$ as

$$\mu_1(\mathbf{h}) = \sum_{k=1}^{L} \alpha_k^2 = \alpha_{\mathrm{D}}^2$$
$$\mu_2(\mathbf{h}) = \alpha_{\mathrm{D}}^4 - \sum_{k=1}^{L} \alpha_k^4$$
$$\mu_3(\mathbf{h}) = 2\alpha_{\mathrm{D}}^6 - 6 \sum_{k=1}^{L} \sum_{n=1}^{L} \alpha_k^2 \alpha_n^4 + 4 \sum_{k=1}^{L} \alpha_k^6. \tag{52}$$

Even when substituting these moments in (32), it is very cumbersome to obtain the expectation of (32) over $\mathbf{h}$ in closed form. Therefore, we introduce another approximation, which consists of evaluating these moments for a particular channel whose random path amplitudes α_k are r.v.s with unitary correlation coefficient, i.e., $\alpha_k = \alpha \cdot \sqrt{\Omega_k}$, $k = 1, \ldots, L$, with α an r.v. representing the channel variability. Clearly, the actual channel multipaths do not vary in unison; it is only a special case that we choose as a representative channel. We stress that this approximation is used only for the computation of the interference term ξ and not for the desired signal, which, on the contrary, is sensitive to channel diversity, and takes advantage of the independence of the paths. In this case, the moments simplify to

$$\mu_1(\mathbf{h}) = \alpha_{\mathrm{D}}^2$$
$$\mu_2(\mathbf{h}) = (1-\beta) \cdot \alpha_{\mathrm{D}}^4$$
$$\mu_3(\mathbf{h}) = (2 - 6\beta + 4\kappa) \cdot \alpha_{\mathrm{D}}^6 \tag{53}$$

where $\beta = \sum_{k=1}^{L} \Omega_k^2$ and $\kappa = \sum_{k=1}^{L} \Omega_k^3$ are constants that depend on the normalized PDP. Now, choosing the constant $z = \sqrt{3\mu_2}$ and substituting (53) into (32), we obtain $\mu_2/z^2 = 1/3$ and $\mu_3/z^3 = (2 - 6\beta + 4\kappa)/(3\sqrt{3}(1-\beta)^{3/2})$. Hence, the approximation (34) for the expectation over the time delays $\mathbf{t}$ is obtained.

Acknowledgment

The authors would like to thank W. M. Gifford and A. Tsankov for their comments and careful reading of the manuscript, and the anonymous reviewers for their valuable comments and constructive suggestions, which have improved the quality of this paper.

References

[1] M. Z. Win and R. A. Scholtz, "Impulse radio: How it works," *IEEE Commun. Lett.*, vol. 2, no. 2, pp. 36–38, Feb. 1998.

[2] ——, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications," *IEEE Trans. Commun.*, vol. 48, no. 4, pp. 679–691, Apr. 2000.

[3] ——, "Characterization of ultra-wide bandwidth wireless indoor communications channel: A communication theoretic view," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1613–1627, Dec. 2002.

[4] M. Z. Win, "A unified spectral analysis of generalized time-hopping spread-spectrum signals in the presence of timing jitter," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1664–1676, Dec. 2002.

[5] M. M. Hämäläinen, V. Hovinen, R. Tesi, J. H. J. Iinatti, and M. Latva-aho, "On the UWB system coexistence with GSM900, UMTS/WCDMA, and GPS," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1712–1721, Dec. 2002.

[6] M. M. Hämäläinen, R. Tesi, and J. Iianatti, "On the UWB system performance studies in AWGN channel with interference in UMTS band," in *Proc. IEEE Conf. Ultra Wideband Syst. Technol.*, May 2002, pp. 321–325.

[7] M. M. Hämäläinen, R. Tesi, J. Iianatti, and V. Hovinen, "On the performance comparison of different UWB data modulation schemes in AWGN channel in the presence of jamming," in *Proc. IEEE Radio Wireless Conf.*, Aug. 2002, pp. 83–86.

[8] S. S. Kolenchery, J. K. Townsend, and J. A. Freebersyser, "A novel impulse radio network for tactical military wireless communications," in *Proc. Mil. Commun. Conf.*, vol. 1, Boston, MA, Oct. 1998, pp. 59–65.

[9] G. D. Weeks, J. K. Townsend, and J. A. Freebersyser, "Performance of hard decision detection for impulse radio," in *Proc. Mil. Commun. Conf.*, vol. 2, Atlantic City, NJ, Oct. 1999, pp. 1201–1206.

[10] J. Conroy, J. L. LoCicero, and D. R. Ucci, "Communication techniques using monopulse waveforms," in *Proc. Mil. Commun. Conf.*, vol. 2, Atlantic City, NJ, Oct. 1999, pp. 1181–1185.

[11] L. Zhao and A. M. Haimovich, "Interference suppression in ultra-wideband communications," in *Proc. Conf. Inf. Sci. Syst.*, vol. 2, Baltimore, MD, Mar. 2001, pp. 759–763.

[12] L. Zhao, A. M. Haimovich, and H. Grebel, "Performance of ultra-wideband communications in the presence of interference," in *Proc. IEEE Int. Conf. Commun.*, vol. 10, Helsinki, Finland, Jun. 2001, pp. 2948–2952.

[13] L. Zhao and A. M. Haimovich, "Performance of ultra-wideband communications in the presence of interference," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1684–1691, Dec. 2002.

[14] *Revision of Part 15 of the Commission's Rules Regarding Ultra-Wideband Transmission Systems, First Report and Order (ET Docket 98-153)*, Federal Communications Commission, adopted Feb. 14, 2002, released Apr. 22, 2002.

[15] G. L. Stuber, *Principles of Mobile Communication*, 2nd ed. Norwell, MA: Kluwer, 2001.

[16] D. Dardari and G. Pasolini, "Simple and accurate models for error probability evaluation of IEEE802.11 DS-SS physical interface in the presence of Bluetooth interference," in *Proc. IEEE Global Telecommun. Conf.*, vol. 1, Taipei, Taiwan, R.O.C., Nov. 2002, pp. 201–206.

[17] M. Moeneclaey, M. V. Bladel, and H. Sari, "Sensitivity of multiple-access techniques to narrowband interference," *IEEE Trans. Commun.*, vol. 49, no. 3, pp. 497–505, Mar. 2001.

[18] L. B. Milstein, S. Davidovici, and D. L. Schilling, "The effect of multiple-tone interfering signals on a direct sequence spread spectrum communication system," *IEEE Trans. Commun.*, vol. COM-30, no. 3, pp. 436–446, Mar. 1982.

[19] R.-H. Dou and L. B. Milstein, "Error probability bounds and approximations for DS spread-spectrum communication systems with multiple tone or multiple access interference," *IEEE Trans. Commun.*, vol. COM-32, no. 5, pp. 493–502, May 1984.

[20] J. D. Choi and W. E. Stark, "Performance analysis of ultra-wideband spread-spectrum communications in narrowband interference," in *Proc. Mil. Commun. Conf.*, vol. 21, Anaheim, CA, Oct. 2002, pp. 1075–1080.

[21] A. Taha and K. M. Chugg, "A theoretical study on the effects of interference UWB multiple access impulse radio," in *Proc. 36th Asilomar Conf. Signals, Syst., Comput.*, vol. 1, Nov. 2002, pp. 728–732.

[22] J. R. Foerster, "The performance of a direct sequence spread ultra-wideband system in the presence of multipath, narrowband interference and multi-user interference," in *Proc. IEEE Conf. Ultra Wideband Syst. Technol.*, Baltimore, MD, May 2002, pp. 87–91.

[23] G. Yue, L. Ge, and S. Li, "Performance of ultra wideband impulse radio in the presence of jamming," in *Proc. Int. Workshop Ultra Wideband Syst.*, Oulu, Finland, Jun. 2003, pp. 1010–1014.

[24] L. Piazzo and J. Romme, "Spectrum control by means of the TH code in UWB systems," in *Proc. IEEE Veh. Technol. Conf.*, vol. 3, Jeju, Korea, Apr. 2003, pp. 1649–1653.

[25] J. Wang and L. B. Milstein, "CDMA overlay situations for microcellular mobile communications," *IEEE Trans. Commun.*, vol. 43, no. 2–4, pp. 603–614, Feb.–Apr. 1995.

[26] ——, "Multicarrier CDMA overlay for ultra-wideband communications," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1664–1669, Oct. 2004.

[27] R. A. Scholtz, R. Weaver, E. Homier, J. Lee, P. Hilmes, A. Taha, and R. Wilson, "UWB radio deployment challenges," in *Proc. IEEE Int. Symp. Pers., Indoor, Mobile Radio Commun.*, vol. 1, London, U.K., Sep. 2000, pp. 620–625.

[28] D. Cassioli, M. Z. Win, and A. F. Molisch, "The ultra-wide bandwidth indoor channel: From statistical model to simulations," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 6, pp. 1247–1257, Aug. 2002.

[29] J. Gil-Pelaez, “Note on the inversion theorem,” *Biometrika*, vol. 38, no. 3/4, pp. 481–482, Dec. 1951.

[30] J. G. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2001.

[31] A. Giorgetti, “Integration of wireless and satellite networks,” Ph.D. thesis, Univ. Bologna, Bologna, Italy, Mar. 2003.

[32] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, 4th ed. San Diego, CA: Academic, 1980.

[33] J. M. Holtzman, “On using perturbation analysis to do sensitivity analysis: Derivatives vs. differences,” in *Proc. 28th IEEE Conf. Decision, Control*, vol. 3, Dec. 1989, pp. 2018–2023.

[34] ——, “A simple, accurate method to calculate spread-spectrum multiple-access error probabilities,” *IEEE Trans. Commun.*, vol. 40, no. 3, pp. 461–464, Mar. 1992.

[35] R. K. Morrow, Jr., “Accurate CDMA BER calculations with low computational complexity,” *IEEE Trans. Commun.*, vol. 46, no. 11, pp. 1413–1417, Nov. 1998.

[36] Y. C. Yoon, “A simple and accurate method of probability of bit-error analysis for asynchronous bandlimited DS-CDMA systems,” *IEEE Trans. Commun.*, vol. 50, no. 4, pp. 656–663, Apr. 2002.

[37] M. Z. Win and J. H. Winters, “Virtual branch analysis of symbol-error probability for hybrid selection/maximal-ratio combining in Rayleigh fading,” *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1926–1934, Nov. 2001.

[38] A. S. Rosenbaum, “Binary PSK error probabilities with multiple cochannel interferences,” *IEEE Trans. Commun.*, vol. COM-18, no. 3, pp. 241–253, Jun. 1970.

[39] L. Piazzo, “UWB EM compatibility and coexistence issues,” in *Proc. 1st Int. Workshop Netw. With UWB*, Rome, Italy, Dec. 2001, pp. 100–104.

Andrea Giorgetti (S’98–M’04) was born in Cesena, Italy, on November 5, 1974. He received the Dr.Ing. degree (*magna cum laude*) in electronic engineering and the Ph.D. degree in electronic engineering and computer science, both from the University of Bologna, Bologna, Italy, in 1999 and 2003, respectively.

Since 2003, he has been with the Istituto di Elettronica e di Ingegneria dell’Informazione e delle Telecomunicazioni (IEIIT), Research Unit at Bologna of the National Research Council (CNR), Bologna, Italy. His research interests include ultrawide bandwidth communications systems, wireless sensor networks, and multiple-antenna systems.

Marco Chiani (M’94–SM’02) was born in Rimini, Italy, in April 1964. He received the Dr. Ing. degree (magna cum laude) in electronic engineering and the Ph.D. degree in electronic and computer science from the University of Bologna, Bologna, Italy, in 1989 and 1993, respectively.

He is a Full Professor at the II Engineering Faculty, University of Bologna, where he is the Chair in Telecommunication. During the summer of 2001, he was a Visiting Scientist at AT&T Research Laboratories, Middletown, NJ. He presently holds a Research Affiliate appointment at the Massachusetts Institute of Technology (MIT), Cambridge. His research interests include wireless communication systems, MIMO systems, wireless multimedia, low-density parity-check codes (LDPCC), and UWB. Under the European research program PROMETHEUS he has worked on short-range millimeter wave communication systems for Advanced Road Transport Telematics. He is also leading the research unit of CNIT/University of Bologna on Joint Source and Channel Coding for wireless video (IST FP6, project Phoenix) and is a consultant to the European Space Agency (ESA-ESOC) for the design and evaluation of error correcting codes based on LDPCC for space CCSDS applications. He has been a consultant for several other telecommunications industries and operators, and engages in active collaboration with many research centers and universities, such as MIT, NASA/JPL, VTT, Thales, Philips, Telecom New Zealand, the University of Canterbury, and the University of Arizona.

Prof. Chiani is actively involved in various communications research activities worldwide. He is the past chair (2002-2004) of the Radio Communications Committee of the IEEE Communication Society and the current Editor of Wireless Communication for the IEEE TRANSACTIONS ON COMMUNICATIONS. He has chaired and organized sessions at several IEEE international conferences, and served on the Technical Program Committees for GLOBECOM 1997, ICC 1999, ICC 2001, and ICC 2002. He was the Chair of the Wireless Communications Symposium at ICC 2004.

Moe Z. Win (S’85–M’87–SM’97–F’04) received the B.S. degree (magna cum laude) from Texas A&M University, College Station, in 1987, and the M.S. degree from the University of Southern California (USC), Los Angeles, in 1989, both in electrical engineering. As a Presidential Fellow at USC, he received both an M.S. degree in applied mathematics and the Ph.D. degree in electrical engineering in 1998.

He is an Associate Professor at the Laboratory for Information & Decision Systems (LIDS), Massachusetts Institute of Technology. Prior to joining LIDS, he spent 5 years at AT&T Research Laboratories and 7 years at the Jet Propulsion Laboratory. His main research interests are the application of mathematical and statistical theories to communication, detection, and estimation problems. Specific current research topics include measurement and modeling of time-varying channels, design and analysis of multiple antenna systems, ultra-wide bandwidth (UWB) communications systems, optical communications systems, and space communications systems.

Professor Win has been involved actively in organizing and chairing a number of international conferences. He served as the Technical Program Chair for the IEEE Communication Theory Symposia of ICC-2004 and Globecom-2000, as well as for the IEEE Conference on Ultra Wideband Systems and Technologies in 2002, Technical Program Vice-Chair for the IEEE International Conference on Communications in 2002, and the Tutorial Chair for the IEEE Semiannual International Vehicular Technology Conference in Fall 2001. He is the current chair and past secretary (2002-2004) for the Radio Communications Committee of the IEEE Communications Society. He currently serves as Area Editor for Modulation and Signal Design and Editor for Wideband Wireless and Diversity, both for IEEE TRANSACTIONS ON COMMUNICATIONS. He served as the Editor for Equalization and Diversity from July 1998 to June 2003 for the IEEE TRANSACTIONS ON COMMUNICATIONS, and as a Guest Editor for the 2002 IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS (Special Issue on Ultra -Wideband Radio in Multiaccess Wireless Communications). He received the International Telecommunications Innovation Award from Korea Electronics Technology Institute in 2002, a Young Investigator Award from the Office of Naval Research in 2003, and the IEEE Antennas and Propagation Society Sergei A. Schelkunoff Transactions Prize Paper Award in 2003. In 2004, he was named Young Aerospace Engineer of the Year by AIAA, and garnered the Fulbright Foundation Senior Scholar Lecturing and Research Fellowship, the Institute of Advanced Study Natural Sciences and Technology Fellowship, the Outstanding International Collaboration Award from the Industrial Technology Research Institute of Taiwan, and the Presidential Early Career Award for Scientists and Engineers from the White House. He was honored with the 2006 IEEE Eric E. Sumner Award for pioneering contributions to ultra-wide band communications science and technology. He is an IEEE Distinguished Lecturer and elected Fellow of the IEEE, cited for contributions to wideband wireless transmission.