

Cooperative Communications with Outage-Optimal Opportunistic Relaying

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Abstract—In this paper, we present simple *opportunistic relaying with decode-and-forward (DaF) and amplify-and-forward (AaF) strategies under an aggregate power constraint. In particular, we consider distributed relay-selection algorithms requiring only local channel knowledge. We show that opportunistic DaF relaying is outage-optimal, that is, it is equivalent in outage behavior to the optimal DaF strategy that employs all potential relays. We further show that opportunistic AaF relaying is outage-optimal among single-relay selection methods and significantly outperforms an AaF strategy based on equal-power multiple-relay transmissions with local channel knowledge. These findings reveal that cooperation offers diversity benefits even when cooperative relays choose not to transmit but rather choose to cooperatively listen; they act as passive relays and give priority to the transmission of a single opportunistic relay. Numerical and simulation results are presented to verify our analysis.*

Index Terms—Cooperative diversity, fading relay channel, outage probability, wireless networks.

I. INTRODUCTION

UTILIZATION of terminals distributed in space can significantly improve the performance of wireless networks [1]–[3]. For example, a pair of neighboring nodes with channel state information (CSI) can cooperatively *beamform* towards the final destination to increase total capacity [2]. Even when CSI is not available or when radio hardware cannot support beamforming, cooperation between the source and a single relay provides improved robustness to wireless fading [3]. Basic results for cooperation are presented in [4]–[6] and references therein.

Scaling cooperation to more than one relay is still an open area of research, despite the recent interest in cooperative communication. One possible approach is the use of distributed space–time coding among participating nodes [7].

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In practice, such code design is quite difficult due to the *distributed* and *ad-hoc* nature of cooperative links, as opposed to codes designed for co-located multiple-input multiple-output (MIMO) systems [8]–[10]. The need and availability of global CSI is fundamental in distributed environments. For example, additional communication is needed for each relay to acquire CSI about other relays (as needed in [11]) or for the destination to acquire CSI between the source and *all* relays (as needed in [12]). Moreover, the number of *useful* antennas (distributed relays) for cooperation is generally unknown and varying. Therefore, coordination among the cooperating nodes is needed prior to the use of a specific space–time coding scheme, typically designed for a fixed number of transmit antennas. Furthermore, it is often assumed in the literature that the superposition of signals transmitted by several relays is *always constructive*.¹

Such assumption requires distributed phased-array techniques (beamforming) and unconventional radios, thereby increasing complexity and cost of each transmitter. Finally, *coherent* reception of multiple-relay (MR) transmissions requires tracking of carrier-phase differences among *several* transmit-receive pairs, which increases the cost of the receiver.

Therefore, simplification of radio hardware in cooperative diversity setups is important. Antenna selection, invented for classical multiple-antenna communications [14]–[18], is one approach to minimize the required *cooperation overhead* and to simultaneously realize the potential benefits of cooperation between multiple relays. In particular, a simple, distributed, single-relay selection algorithm was proposed for slow fading wireless relay channels [19]. This single-relay *opportunistic* selection provides no performance loss from the perspective of diversity–multiplexing gain tradeoff, compared to schemes that rely on distributed space–time coding.

In this paper, we present single-selection—*opportunistic*—relaying with decode-and-forward (DaF) and amplify-and-forward (AaF) strategies and analyze their outage probability under an aggregate power constraint.² The motivation behind imposing the aggregate power constraint is threefold: (i) transmission power is a network resource that affects both the lifetime of the network with battery-operated terminals and the scalability of the network; (ii) regulatory agencies may limit total transmission power due to the fact that each transmission can cause *interference* to the others in the network; and (iii) cooperative diversity benefits can be exploited even when relays *do not* transmit (and therefore, do not add transmission

¹This case includes Gaussian relay channels where propagation coefficients are assumed to be real numbers [13].

²The DaF strategy is also known as regenerative processing.

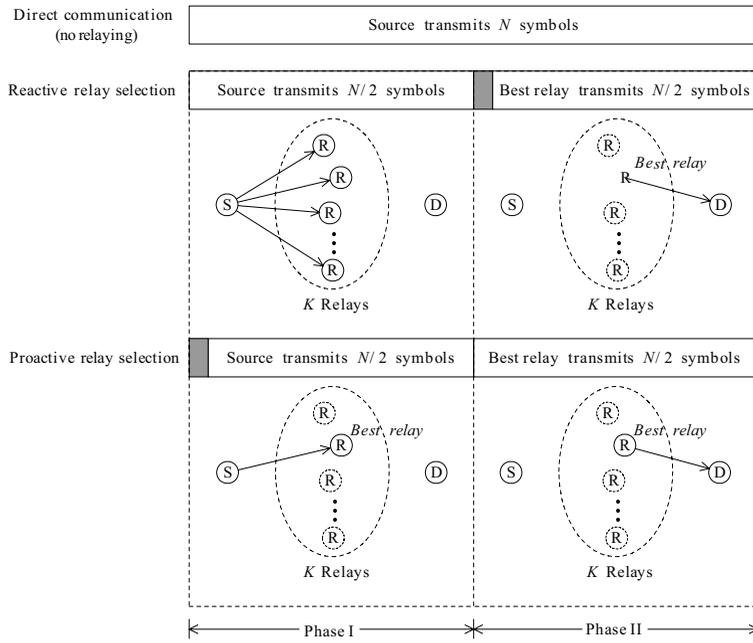


Fig. 1. A half-duplex dual-hop communication scenario: the source and destination are blocked or have *poor* connection. Opportunistic relay selection can be performed *proactively* before the source transmission or *reactively* after the source transmission. The shaded band indicates when relay selection occurs.

energy into the network). We consider both *reactive* and *proactive* relay selection depending on whether the relay selection is performed after or before the source transmission.

The contributions of this paper are as follows.

- We propose simple opportunistic relaying schemes with DaF and AaF strategies, which can be performed in a distributed manner without requiring *global* CSI at each relay or at a central controller in the network, thereby reducing the required cooperation overhead.
- We show that both reactive and proactive opportunistic DaF relaying are outage-optimal, that is, they are equivalent in outage behavior to the optimal DaF strategy that employs all potential relays.
- We show that opportunistic AaF relaying is outage-optimal among single-relay selection methods. Additionally, opportunistic AaF significantly outperforms an AaF strategy based on *equal-power* MR transmissions when only local CSI is available.
- Proactive opportunistic relay selection allows all relays, except a single opportunistic relay, to enter an idle mode during the source transmission, thereby reducing the reception energy cost in the network.

These results reveal that relays are useful even when they do not actively transmit, provided that they adhere to the “opportunistic” cooperation rule and give priority to the “best” available relay. The simplicity of our protocol allows immediate implementation in a custom radio hardware.³

The remainder of the paper is organized as follows. In Section II, we present the basic protocols examined in this work and in Sections III and IV, we analyze DaF and AaF strategies, respectively. Finally, conclusions are given in Section VI.

II. MODELS AND PROTOCOLS

We consider a half-duplex dual-hop communication scenario in a cluttered environment depicted in Fig. 1, where the direct path between the source and destination is blocked by an intermediate wall, while relays are located at the periphery of the obstacle (around-the-corner). The relays can communicate with both endpoints (source and destination). During the first hop, the source (without exploiting any CSI) transmits $N/2$ symbols and the relays listen, while during the second hop, the relays forward a version of the received signal using the same number of symbols.⁴ The channel is assumed to remain constant during the two hops (at least N -symbol coherence time) with Rayleigh fading. We further consider a source power constraint

$$\mathcal{P}_{\text{source}} = \zeta \mathcal{P}_{\text{tot}} \quad (1)$$

and an aggregate relay power constraint

$$\mathcal{P}_{\text{relay}} = \sum_{k=1}^K \mathcal{P}_k = (1 - \zeta) \mathcal{P}_{\text{tot}} \quad (2)$$

where K is the number of relays, \mathcal{P}_{tot} is the total end-to-end (i.e., source-relay-destination) transmission power, $\mathcal{P}_{\text{source}}$ is the transmission power of the source, \mathcal{P}_k , $k = 1, 2, \dots, K$, is the transmission power of the k th relay, and $\mathcal{P}_{\text{relay}}$ is the aggregate relay power allocated to the set $\mathcal{S}_{\text{relay}} = \{1, 2, \dots, K\}$ of K relays. Note that if the k th relay does not participate in relaying, \mathcal{P}_k is equal to zero. Also, $\zeta \in (0, 1]$ and $(1 - \zeta) \in [0, 1)$ denote the fractions of the total end-to-end

⁴If the source is allowed to transmit different symbols during the second hop, one channel degree of freedom would not be wasted and the spectral efficiency can be improved [4], [20]. However, in this paper, we are interested in finding the optimal strategy for relay transmissions and hence, simplify their operation.

³An implementation example can be found in [19].

power \mathcal{P}_{tot} allocated to the source transmission and overall relay transmission, respectively.

It should be noted that the optimal power allocation across the source and relays depends on CSI knowledge and can be $\mathcal{P}_{\text{source}} \neq \mathcal{P}_{\text{relay}}$ [21]. However, this is feasible only when global CSI about the whole network (including channel states between the relays and destination) is available at the source. In this work, we do not assume CSI at the source. Furthermore, optimal power allocation becomes more important when there is a good link between the source and final destination. None of these conditions are applicable to our study. In fact, our main focus is not just optimal power allocation but a more general question of what relays should do optimally—*re-transmit or not*.

We now provide the model for the received signal in a link ($A \rightarrow B$) between two nodes “A” and “B”:

$$y_B = \alpha_{AB} x_A + n_B \quad (3)$$

where x_A is the signal transmitted at the node A, $\alpha_{AB} \sim \mathcal{CN}(0, \Omega_{AB})$ is the channel gain between the link $A \rightarrow B$, and $n_B \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) at the node B.⁵ For each link, let $\gamma_{AB} \triangleq |\alpha_{AB}|^2$ be the instantaneous squared channel strength, which obeys an exponential distribution with hazard rate $1/\Omega_{AB}$, denoted by $\gamma_{AB} \sim \Upsilon(1/\Omega_{AB})$. The probability density function (p.d.f.) of γ_{AB} is given by

$$p_{\gamma_{AB}}(x) = \frac{1}{\Omega_{AB}} \exp(-x/\Omega_{AB}), \quad x \geq 0. \quad (4)$$

If the node A is the source, then $\mathbb{E}\{|x_A|^2\} = \mathcal{P}_{\text{source}}$. Similarly, if the node A is the k th relay, then $\mathbb{E}\{|x_A|^2\} = \mathcal{P}_k$. Specifically, for each relay $k \in \mathcal{S}_{\text{relay}}$, we designate a link from the source to the k th relay by $S \rightarrow k$ and a link from the k th relay to the destination by $k \rightarrow D$. For the links $S \rightarrow k$ and $k \rightarrow D$, the received average signal-to-noise ratios (SNRs) are equal to $\eta_{Sk} \triangleq \Omega_{Sk} \mathcal{P}_{\text{source}}/N_0$ and $\eta_{kD} \triangleq \Omega_{kD} \mathcal{P}_k/N_0$, respectively.⁶

To reduce overhead and simplify protocol implementation, cooperation is coordinated only every N symbols. As shown in Fig. 1, we consider two modes of coordination: (i) *reactive* coordination among DaF relays and (ii) *proactive* coordination among DaF or AaF relays. In a reactive mode, relays that successfully decode the message participate in cooperation, whereas in a proactive mode, specific relays that are selected prior to the source transmission participate in cooperation.

Relay selection can be performed without requiring global CSI at each relay or at a central controller in the network. One possible approach is to use the method of distributed timers proposed in [19], where each relay estimates its own instantaneous channel paths towards the source and the destination.

⁵ $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex circularly symmetric Gaussian distribution with mean μ and variance σ^2 . Similarly, $\mathcal{N}_m(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes a complex m -variate Gaussian distribution with a mean vector $\boldsymbol{\mu} \in \mathbb{C}^m$ and a covariance matrix $\boldsymbol{\Sigma} \in \mathbb{C}^{m \times m}$. Note that a specific time index is dropped in (3).

⁶We consider a scenario in which the channel gains for all links are statistically independent. In addition, since we consider a different average channel gain $\mathbb{E}\{\gamma_{AB}\} = \Omega_{AB}$ for each link, the noise variances at all nodes are normalized to N_0 without loss of generality. Throughout the paper, we use the term ‘SNR’ to refer to instantaneous SNR. The term ‘average SNR’ is explicitly used to describe the SNR averaged over the fading ensemble.

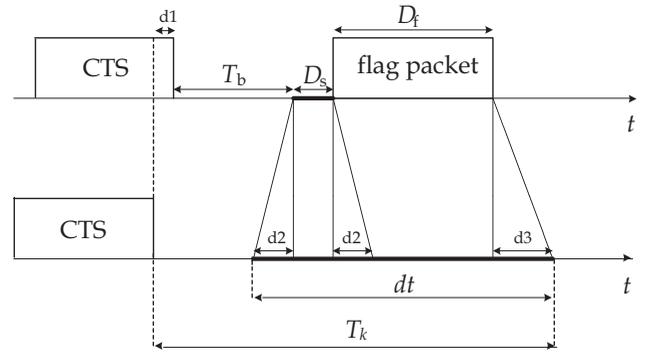


Fig. 2. Distributed relay selection: Upon reception of CTS from destination, each relay starts its own timer T_k with a metric of its *own* channel conditions. The relay with the highest metric will have its timer T_b expire first and broadcast a flag packet, notifying its availability for relaying to the rest of the terminals.

This can be accomplished by listening pilot signals transmitted from the source (Ready-To-Send or RTS) and transmitted from the destination (Clear-to-Send or CTS). Upon receiving CTS, each relay k then starts a timer T_k whose duration is inversely proportional to a metric depending only on its *own* channel gains towards source $|\alpha_{Sk}|$ and destination $|\alpha_{kD}|$ (see Fig. 2). The timer T_b of the “best” relay b expires first and a flag packet with duration D_f notifies the rest of the network about its availability.⁷

Since all cooperative relays *compete* to access the wireless medium according to their own channel conditions, there is a finite probability that *any* two relays have their timer expire within the same time interval dt and transmit their flag packets (Fig. 2). In that case, the destination can assess that more than one relays are possible candidates. Such probability depends on 1) the propagation delays $d1, d3$ from destination to relays,⁸ 2) propagation delays $d2$ between the relays, 3) radio listen-to-transmit switch time D_s and 4) duration D_f of the flag packet when relays cannot listen to each other (the case of *hidden* relay terminals). That probability was analytically calculated for any type of wireless fading statistics in [19] and [22]. It was shown that opportunistic relay selection can be completed within a fraction of the channel coherence time. Additional details regarding the above distributed relay selection protocol without global CSI and implementation examples with low-cost radios can be found in [19].

III. DECODE-AND-FORWARD RELAYING

A. Reactive DaF

1) *Reactive Multiple-Relay DaF*: In a reactive MR scheme with DaF strategy, the relays that successfully receive the message during the first phase regenerate and transmit it during the second phase, possibly through a distributed space-time code [7]. The transmissions during the second phase are performed only by a subset \mathcal{D} of K relays, defined by

$$\mathcal{D} \triangleq \left\{ k \in \mathcal{S}_{\text{relay}} : \frac{1}{2} \log_2 \left(1 + \zeta \gamma_{Sk} \frac{\mathcal{P}_{\text{tot}}}{N_0} \right) \geq R \right\} \quad (5)$$

⁷Note that no explicit time-synchronization protocol is required among the relays.

⁸ $d1$ is one-way propagation delay and $d3$ includes round-trip propagation delay.

where R denotes the end-to-end spectral efficiency in bps/Hz. In (5), the decoding at relay k is assumed to be successful if $\frac{1}{2} \log_2 (1 + \zeta \gamma_{S_k} \mathcal{P}_{\text{tot}}/N_0) \geq R$, i.e., no outage event happens during the first phase [3], [7]. Since communication happens in two half-duplex hops, the required spectral efficiency per hop is equal to $2R$ so that the end-to-end spectral efficiency is R , which is comparable to direct non-cooperative communication.

Let $\mathcal{D}_\ell \subseteq \mathcal{S}_{\text{relay}}$ be a decoding subset with ℓ relays (i.e., cardinality $|\mathcal{D}_\ell| = \ell$). Then, we have

$$\begin{aligned} \Pr\{\mathcal{D}_\ell\} &= \prod_{i \in \mathcal{D}_\ell} \Pr\{\gamma_{S_i} \geq \kappa_1\} \prod_{j \notin \mathcal{D}_\ell} \Pr\{\gamma_{S_j} \leq \kappa_1\} \\ &= \prod_{i \in \mathcal{D}_\ell} e^{-\kappa_1/\Omega_{S_i}} \prod_{j \notin \mathcal{D}_\ell} \left(1 - e^{-\kappa_1/\Omega_{S_j}}\right) \end{aligned} \quad (6)$$

where $\kappa_1 = \frac{2^{2R}-1}{\zeta \mathcal{P}_{\text{tot}}/N_0}$. The outage probability for reactive MR transmissions with DaF strategy can be written as

$$P_{\text{MR-DaF}}^{(\text{react})}(\text{outage}) = \sum_{\ell=0}^K \sum_{\mathcal{D}_\ell} \Pr\{\text{outage} | \mathcal{D}_\ell\} \Pr\{\mathcal{D}_\ell\} \quad (7)$$

where the second summation is over all $\binom{K}{\ell}$ different decoding subsets with exactly ℓ successfully decodable relays.⁹ In (7), the conditional outage probability is given by

$$\Pr\{\text{outage} | \mathcal{D}_\ell\} = \Pr\left\{\frac{1}{2} \log_2 \left(1 + \sum_{k \in \mathcal{D}_\ell} \gamma_{kD} \frac{\mathcal{P}_k}{N_0}\right) < R\right\} \quad (8)$$

with $\sum_{k \in \mathcal{D}_\ell} \mathcal{P}_k = \mathcal{P}_{\text{relay}}$.

Let $\{\varphi_i(\mathcal{D}_\ell)\}_{i=1}^\ell = \{\eta_{kD}\}_{k \in \mathcal{D}_\ell}$ and

$$\mathbf{A}(\mathcal{D}_\ell) = \text{diag}(\varphi_1(\mathcal{D}_\ell), \varphi_2(\mathcal{D}_\ell), \dots, \varphi_\ell(\mathcal{D}_\ell)).$$

Then, using Theorem 2 in Appendix A, we obtain the conditional outage probability $\Pr\{\text{outage} | \mathcal{D}_\ell\}$ as (9), shown at the bottom of the page, where $\varrho(\mathbf{A}(\mathcal{D}_\ell))$ is the number of distinct diagonal elements of $\mathbf{A}(\mathcal{D}_\ell)$, $\varphi_{(1)}(\mathcal{D}_\ell) > \varphi_{(2)}(\mathcal{D}_\ell) > \dots > \varphi_{(\varrho(\mathbf{A}(\mathcal{D}_\ell)))(\mathcal{D}_\ell)}$ are the distinct diagonal elements in decreasing order, $\tau_i(\mathbf{A}(\mathcal{D}_\ell))$ is the multiplicity of $\varphi_{(i)}(\mathcal{D}_\ell)$, and $\mathcal{X}_{i,j}(\mathbf{A}(\mathcal{D}_\ell))$ is the (i,j) th characteristic coefficient of

⁹The equality in (7) is due to the total probability theorem over disjoint sets \mathcal{D}_ℓ that partition the sample space. Note that there are 2^K possible decoding subsets for K relays, including \mathcal{D}_0 , i.e., the set with no decodable relay during the first hop of the protocol.

$\mathbf{A}(\mathcal{D}_\ell)$ [23]. Combining (6), (7), and (9) gives (10), shown at the bottom of the page.

2) *Reactive Opportunistic DaF*: The following theorem establishes the fact that opportunistic DaF relaying is outage-optimal, that is, it is equivalent in outage behavior to the optimal DaF strategy that employs all potential relays.

Theorem 1: For a reactive DaF relaying scheme with the aggregate power constraint (2), choosing the “best” relay $b_{\text{react-DaF}}^*$ that maximizes the instantaneous channel strength between the links $k \rightarrow D$ for all $k \in \mathcal{D}_\ell$, that is,

$$b_{\text{react-DaF}}^* = \arg \max_{k \in \mathcal{D}_\ell} \gamma_{kD} \quad (11)$$

is outage-optimal.

Proof: For any reactive DaF relaying scheme with the aggregate power constraint (2), the received SNR at the destination is upper bounded as

$$\begin{aligned} \sum_{k \in \mathcal{D}_\ell} \gamma_{kD} \frac{\mathcal{P}_k}{N_0} &\leq \sum_{k \in \mathcal{D}_\ell} \left(\max_{k \in \mathcal{D}_\ell} \gamma_{kD}\right) \frac{\mathcal{P}_k}{N_0} \\ &= (1 - \zeta) \max_{k \in \mathcal{D}_\ell} \gamma_{kD} \frac{\mathcal{P}_{\text{tot}}}{N_0}. \end{aligned} \quad (12)$$

Therefore, the conditional outage probability in (8) for any reactive DaF relaying scheme is lower bounded as follows:

$$\begin{aligned} &\Pr\{\text{outage} | \mathcal{D}_\ell\} \\ &\geq \Pr\left\{\frac{1}{2} \log_2 \left(1 + (1 - \zeta) \max_{k \in \mathcal{D}_\ell} \gamma_{kD} \frac{\mathcal{P}_{\text{tot}}}{N_0}\right) < R\right\} \end{aligned} \quad (13)$$

$$= \prod_{k \in \mathcal{D}_\ell} \Pr\{\gamma_{kD} < \kappa_2\} \quad (14)$$

where $\kappa_2 = \frac{2^{2R}-1}{(1-\zeta)\mathcal{P}_{\text{tot}}/N_0}$. Since the maximum received SNR in (12) and the minimum conditional outage probability in the right-hand side of (13) are achieved by the single opportunistic relay-selection rule (11), we complete the proof of its outage optimality. \square

Note that (14) states simply that if the “best” relay fails, then all relays in \mathcal{D}_ℓ fail because the “best” relay has the strongest path $\gamma_{b_{\text{react-DaF}}^*D}$ between the links $k \rightarrow D$ for all $k \in \mathcal{D}_\ell$. We remark that the minimization of (14) holds for any power allocation ζ . For quasi-static fading environments, a simple method can be devised to select the relay with the maximum channel strength $\gamma_{b_{\text{react-DaF}}^*D}$ in a distributed manner similar to the work in [19] and [22].

$$\Pr\{\text{outage} | \mathcal{D}_\ell\} = 1 - \sum_{i=1}^{\varrho(\mathbf{A}(\mathcal{D}_\ell))} \tau_i(\mathbf{A}(\mathcal{D}_\ell)) \sum_{j=1}^{j-1} \left[\frac{\mathcal{X}_{i,j}(\mathbf{A}(\mathcal{D}_\ell))}{k!} \left(\frac{2^{2R}-1}{\varphi_{(i)}(\mathcal{D}_\ell)}\right)^k \exp\left(-\frac{2^{2R}-1}{\varphi_{(i)}(\mathcal{D}_\ell)}\right) \right] \quad (9)$$

$$\begin{aligned} P_{\text{MR-DaF}}^{(\text{react})}(\text{outage}) &= \sum_{\ell=0}^K \sum_{\mathcal{D}_\ell} \left[\prod_{i \in \mathcal{D}_\ell} e^{-\kappa_1/\Omega_{S_i}} \prod_{j \notin \mathcal{D}_\ell} \left(1 - e^{-\kappa_1/\Omega_{S_j}}\right) \right. \\ &\quad \left. \times \left\{ 1 - \sum_{i=1}^{\varrho(\mathbf{A}(\mathcal{D}_\ell))} \tau_i(\mathbf{A}(\mathcal{D}_\ell)) \sum_{j=1}^{j-1} \frac{\mathcal{X}_{i,j}(\mathbf{A}(\mathcal{D}_\ell))}{k!} \left(\frac{2^{2R}-1}{\varphi_{(i)}(\mathcal{D}_\ell)}\right)^k \exp\left(-\frac{2^{2R}-1}{\varphi_{(i)}(\mathcal{D}_\ell)}\right) \right\} \right] \end{aligned} \quad (10)$$

Using (6) and (7) in conjunction with (14) for the opportunistic relay-selection rule (11), we obtain the outage probability for reactive opportunistic DaF relaying as

$$\begin{aligned} P_{\text{Opp-DaF}}^{\text{(react)}}(\text{outage}) &= \sum_{\ell=0}^K \sum_{\mathcal{D}_\ell} \left[\prod_{i \in \mathcal{D}_\ell} e^{-\frac{\kappa_1}{\Omega_{Si}}} \left(1 - e^{-\frac{\kappa_2}{\Omega_{iD}}}\right) \prod_{j \notin \mathcal{D}_\ell} \left(1 - e^{-\frac{\kappa_1}{\Omega_{Sj}}}\right) \right] \\ &= \prod_{k=1}^K \left[1 - \exp \left\{ -\frac{2^{2R}-1}{\mathcal{P}_{\text{tot}}/N_0} \left(\frac{1}{\zeta \Omega_{Sk}} + \frac{1}{(1-\zeta) \Omega_{kD}} \right) \right\} \right] \end{aligned} \quad (15)$$

where the last equality follows from the multinomial equality

$$\begin{aligned} \prod_{k=1}^K (1 - a_k b_k) &= \prod_{k=1}^K [a_k (1 - b_k) + (1 - a_k)] \\ &= \sum_{\ell=0}^K \sum_{\substack{\mathcal{S}_\ell \subseteq \{1, \dots, K\} \\ |\mathcal{S}_\ell| = \ell}} \left[\prod_{i \in \mathcal{S}_\ell} a_i (1 - b_i) \prod_{j \notin \mathcal{S}_\ell} (1 - a_j) \right]. \end{aligned} \quad (16)$$

Note that (15) implies that opportunistic DaF relaying is in outage only when all of the relays are in outage. In this case, no other schemes can communicate reliably at rate R . Hence, the reactive opportunistic DaF relaying is optimal under the aggregate relay power constraint (2) in a sense that it minimizes the end-to-end outage probability.

In contrast to our single-relay opportunistic rule, one may consider selection of the relay that maximizes the *average* forward channel gain among the decoding set (see, e.g., [24], [25]):

$$\begin{aligned} b_{\text{react-DaF}}^* &= \arg \max_{k \in \mathcal{D}_\ell} \mathbb{E} \{ \gamma_{kD} \} \\ &= \arg \max_{k \in \mathcal{D}_\ell} \Omega_{kD}. \end{aligned} \quad (17)$$

B. Proactive DaF

It might seem that selecting a single relay *before* information is transmitted from the source, could potentially result in degraded performance. On the other hand, selecting a single relay for information forwarding simplifies the receiver design and the overall network operation, since proactive selection is equivalent to *routing*. In what follows, we show that such choice on protocol design incurs no performance loss.

1) *Proactive Opportunistic DaF*: In proactive opportunistic relaying, the “best” relay $b_{\text{proact-DaF}}^*$ is chosen prior to the source transmission among a collection of K possible candidates in a distributed fashion that requires each relay to know its own instantaneous signal strength (but not phase) between the links $S \rightarrow k$ and $k \rightarrow D$, for each relay $k \in \mathcal{S}_{\text{relay}}$.¹⁰ The “best” relay $b_{\text{proact-DaF}}^*$ is chosen to maximize the minimum of the weighted channel strengths between the links $S \rightarrow k$ and $k \rightarrow D$ for all $k \in \mathcal{S}_{\text{relay}}$.¹¹

$$b_{\text{proact-DaF}}^* = \arg \max_{k \in \mathcal{S}_{\text{relay}}} W_k^{(\text{DaF})} \quad (18)$$

¹⁰Relay selection can be accomplished using a method of distributed timers described in Section II, without requiring global CSI.

¹¹Instead of the minimum, the harmonic mean of two path strengths has been also considered in [19].

where $W_k^{(\text{DaF})} = \min \{ \zeta \gamma_{Sk}, (1 - \zeta) \gamma_{kD} \}$.

In this case, communication through the “best” opportunistic relay fails due to outage when either of the two hops (from the source to the best relay or from the best relay to destination) fail. Recall that

$$W_k^{(\text{DaF})} \sim \Upsilon \left(\frac{1}{\zeta \Omega_{Sk}} + \frac{1}{(1-\zeta) \Omega_{kD}} \right) \quad (19)$$

which follows from the fact that the minimum of two independent exponential random variables (r.v.’s) is again an exponential r.v. with a hazard rate equal to the sum of the two hazard rates. From (18) and (19), we obtain the outage probability for proactive opportunistic DaF relaying as follows:

$$\begin{aligned} P_{\text{Opp-DaF}}^{\text{(proact)}}(\text{outage}) &= \Pr \left\{ W_{b_{\text{proact-DaF}}^*}^{(\text{DaF})} < \frac{2^{2R}-1}{\mathcal{P}_{\text{tot}}/N_0} \right\} \\ &= \Pr \left\{ \max_{k \in \mathcal{S}_{\text{relay}}} W_k^{(\text{DaF})} < \frac{2^{2R}-1}{\mathcal{P}_{\text{tot}}/N_0} \right\} \\ &= \prod_{k=1}^K \Pr \left\{ W_k^{(\text{DaF})} < \frac{2^{2R}-1}{\mathcal{P}_{\text{tot}}/N_0} \right\} \\ &= \prod_{k=1}^K \left[1 - \exp \left\{ -\frac{2^{2R}-1}{\mathcal{P}_{\text{tot}}/N_0} \left(\frac{1}{\zeta \Omega_{Sk}} + \frac{1}{(1-\zeta) \Omega_{kD}} \right) \right\} \right]. \end{aligned} \quad (20)$$

It is worth remarking that the outage probability in (20) agrees *exactly* with that in (15) for reactive opportunistic relays with DaF strategy. Since we have shown in the previous section that reactive opportunistic relaying is outage-optimal, the proactive opportunistic “*max-min*” relay selection in (18) is also outage-optimal. Moreover, proactive coordination requires a smaller cooperation overhead in reception energy since all relays, except a single opportunistic relay, can enter an idle mode during the first hop of the protocol. Therefore, our proactive strategy can be viewed as energy-efficient routing in the network. In contrast, the reactive schemes require all relays to receive information during the first hop and therefore, cooperation overhead in reception energy scales proportionally with the network size.

IV. AMPLIFY-AND-FORWARD RELAYING

For the case of AaF, each relay normalizes the signal y_k , $k \in \mathcal{S}_{\text{relay}}$, received during the first phase of the protocol (Fig. 1) and transmits $x_k = \sqrt{\mathcal{P}_k} \frac{y_k}{\sqrt{\mathbb{E}\{|y_k|^2\}}}$ during the second phase of the protocol. The received signal y_D at the destination after the second phase of the protocol can be written as

$$y_D = h x_S + n_D \quad (21)$$

where

$$h = \frac{1}{\check{\sigma}} \sum_{k=1}^K \sqrt{\frac{\mathcal{P}_k}{\Omega_{Sk} \mathcal{P}_{\text{source}} + N_0}} \alpha_{Sk} \alpha_{kD} \quad (22)$$

with

$$\check{\sigma}^2 = 1 + \sum_{k=1}^K \frac{\mathcal{P}_k |\alpha_{kD}|^2}{\Omega_{Sk} \mathcal{P}_{\text{source}} + N_0}. \quad (23)$$

When conditioned on the channel gains from relays to the destination $\{\alpha_{kD}\}_{k=1}^K$, the noise parameter n_D at the destination is found to be a zero-mean complex Gaussian random variable with variance N_0 . From (21), it is easy to find that the mutual information for the AaF strategy with K relays subject to the power constraint (2) is given by

$$\begin{aligned} \mathcal{I}_{\text{MR-AaF}} &= \frac{1}{2} \log_2 \left(1 + |h|^2 \frac{\mathcal{P}_{\text{source}}}{N_0} \right) \\ &= \frac{1}{2} \log_2 \left\{ 1 + \frac{\left| \sum_{k=1}^K \sqrt{\frac{\mathcal{P}_k}{\Omega_{Sk} \mathcal{P}_{\text{source}} + N_0}} \alpha_{Sk} \alpha_{kD} \right|^2 \mathcal{P}_{\text{source}}}{\left(1 + \sum_{k=1}^K \frac{\mathcal{P}_k |\alpha_{kD}|^2}{\Omega_{Sk} \mathcal{P}_{\text{source}} + N_0} \right) N_0} \right\}. \end{aligned} \quad (24)$$

A. Opportunistic AaF

From (24), we see that the maximum mutual information with a single-relay selection, i.e., the mutual information for opportunistic AaF relaying is

$$\mathcal{I}_{\text{Opp-AaF}} = \max_{k \in \mathcal{S}_{\text{relay}}} \frac{1}{2} \log_2 \left\{ 1 + \frac{\gamma_{Sk} \gamma_{kD}}{\frac{\zeta}{1-\zeta} \Omega_{Sk} + \frac{N_0}{\mathcal{P}_{\text{relay}}} + \gamma_{kD}} \frac{\mathcal{P}_{\text{source}}}{N_0} \right\}. \quad (25)$$

Hence, for opportunistic relaying, the “best” relay b_{AaF}^* among K relays in $\mathcal{S}_{\text{relay}}$ is chosen proactively to maximize the mutual information (or to minimize the outage probability) as follows:

$$b_{\text{AaF}}^* = \arg \max_{k \in \mathcal{S}_{\text{relay}}} W_k^{(\text{AaF})} \quad (26)$$

where

$$W_k^{(\text{AaF})} = \frac{\gamma_{Sk} \gamma_{kD}}{\frac{\zeta}{1-\zeta} \left(1 + \frac{1}{\eta_{Sk}} \right) \Omega_{Sk} + \gamma_{kD}}. \quad (27)$$

Note that individual relays do not need to acquire CSI about the links of other relays and hence, the opportunistic relay (26) can also be selected in a distributed manner [19], [22].

Then, using Theorem 3 in Appendix B, we obtain the outage probability for opportunistic AaF relaying as (28), shown at the bottom of the page.

V. NUMERICAL AND SIMULATION RESULTS

In this section, we give numerical examples of the outage probability as a function of $\mathcal{P}_{\text{tot}}/N_0$ with power allocation $\zeta = 0.5$. The optimal power allocation ζ is feasible, only when the source has knowledge of the overall network topology in terms of the average channel gains Ω_{Sk} and Ω_{kD} for all participating relays $k \in \mathcal{S}_{\text{relay}}$. However, this is impractical since

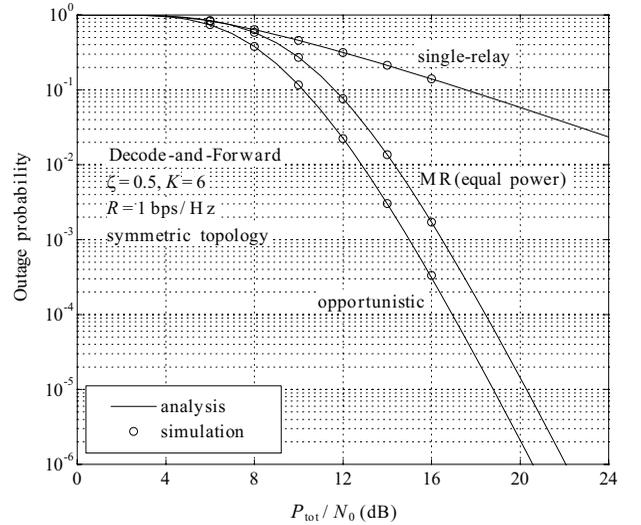


Fig. 3. Outage probability as a function of $\mathcal{P}_{\text{tot}}/N_0$ for the DaF strategy at the end-to-end spectral efficiency $R = 1$ bps/Hz in symmetric channels. $\zeta = 0.5$, $K = 6$, and $\Omega_{Sk} = \Omega_{kD} = 1$, $k = 1, 2, \dots, 6$. Opportunistic relaying is compared with reactive equal-power MR transmissions and single-relay selection based on the maximum average channel gain $\max_{k \in \mathcal{D}_\ell} \Omega_{kD}$.

such knowledge requires considerable overhead. Therefore, the equal-power allocation to the source and the opportunistic relay, i.e., $\zeta = 0.5$ is a natural choice. We further quantify the performance difference between $\zeta = 0.5$ and optimal choice of ζ that requires global CSI at the transmitter and the relay. Our results accommodate both symmetric and asymmetric topologies for DaF or AaF strategy.

A. Decode-and-Forward Relaying

Fig. 3 shows the outage probability as a function of $\mathcal{P}_{\text{tot}}/N_0$ for the DaF strategy with 6 relays ($K = 6$) at the end-to-end spectral efficiency $R = 1$ bps/Hz in symmetric channels with $\Omega_{Sk} = \Omega_{kD} = 1$, $k = 1, 2, \dots, 6$. In this figure, we show the performance of (i) proactive opportunistic DaF relaying, (ii) reactive DaF relaying with equal-power MR transmissions, and (iii) reactive DaF relaying via single-relay selection based on the maximum average channel gain $\max_{k \in \mathcal{D}_\ell} \Omega_{kD}$. Fig. 4 compares the same scenarios in asymmetric channels with $\{\Omega_{Sk}\}_{k=1}^K = \{\Omega_{kD}\}_{k=1}^K = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$. Note that for the symmetric case, single-relay selection based on the average channel gains amounts to selecting just one successful relay randomly (since all relays in the decoding subset \mathcal{D}_ℓ have the same average channel gain to the destination) and transmitting with full relaying power $\mathcal{P}_{\text{relay}}$. Note also that under limited channel knowledge at each relay, the optimal

$$\begin{aligned} P_{\text{Opp-AaF}}(\text{outage}) &= \Pr \left\{ W_{b_{\text{AaF}}^*}^{(\text{AaF})} < \kappa_1 \right\} \\ &= \Pr \left\{ \max_{k \in \mathcal{S}_{\text{relay}}} W_k^{(\text{AaF})} < \kappa_1 \right\} \\ &= \prod_{k=1}^K \left[1 - \frac{1}{\Omega_{kD}} \int_0^\infty \exp \left\{ -\frac{2^{2R} - 1}{\eta_{Sk}} \left(1 + \frac{\zeta}{1-\zeta} \left(1 + \frac{1}{\eta_{Sk}} \right) \frac{\Omega_{Sk}}{z} \right) - \frac{z}{\Omega_{kD}} \right\} dz \right]. \end{aligned} \quad (28)$$

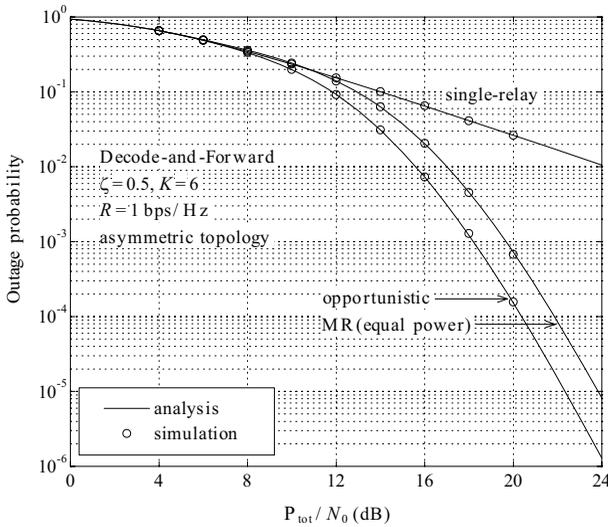


Fig. 4. Outage probability as a function of $\mathcal{P}_{\text{tot}}/N_0$ for the DaF strategy at the end-to-end spectral efficiency $R = 1$ bps/Hz in asymmetric channels. $\zeta = 0.5$, $K = 6$, and $\{\Omega_{S_k}\}_{k=1}^K = \{\Omega_{kD}\}_{k=1}^K = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$. Opportunistic relaying is compared with reactive equal-power MR transmissions and single-relay selection based on the maximum average channel gain $\max_{k \in \mathcal{D}_\ell} \Omega_{kD}$.

power allocation for MR transmission with DaF strategy is infeasible and equal power for the decoding subset \mathcal{D}_ℓ , i.e., $\mathcal{P}_k = \mathcal{P}_{\text{relay}}/\ell$ for all $k \in \mathcal{D}_\ell$ is a reasonable solution in reactive DaF relaying.

Both figures show that opportunistic relaying, despite its simplicity, provides a gain in $\mathcal{P}_{\text{tot}}/N_0$ on the order of 2 dB relative to MR transmission with DaF strategy. This finding reveals that cooperative diversity gains do not necessarily arise from simultaneous transmissions but instead, resilience to fading arises from the availability of several potential paths towards the destination. It is therefore beneficial to select the “best” one.¹² In contrast to single *opportunistic* relay selection, Figs. 3 and 4 also show that single-relay selection based on *average* channel gains incurs a substantial penalty loss. This is due to the fact that selecting a relay with average channel gains removes potential selection diversity benefits.

Fig. 5 and Fig. 6 show the performance of opportunistic DaF relaying as a function of ζ for the symmetric and asymmetric scenarios, respectively. It is shown that $\zeta = 0.5$ is optimal. This can be verified analytically by differentiating (15) or (20). We note however that for the general case of $\Omega_{S_k} \neq \Omega_{kD}$, for any relay k , optimal ζ^* will be different from 0.5.

B. Amplify-and-Forward Relaying

The outage probability as a function of $\mathcal{P}_{\text{tot}}/N_0$ for the AaF strategy with 6 relays ($K = 6$) at the end-to-end spectral efficiency $R = 1$ bps/Hz is plotted in Figs. 7 and 8 for the symmetric channels ($\Omega_{S_k} = \Omega_{kD} = 1$, $k = 1, 2, \dots, 6$) and asymmetric channels ($\{\Omega_{S_k}\}_{k=1}^K = \{\Omega_{kD}\}_{k=1}^K = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$), respectively. We

¹²The main difficulty here is to have the network as a whole entity cooperate in order to rapidly discover the best path with minimal overhead. Ideas on how such selection can be performed in a distributed manner for slow fading environments can be found in [19], where actual implementations with low-cost radios were demonstrated.

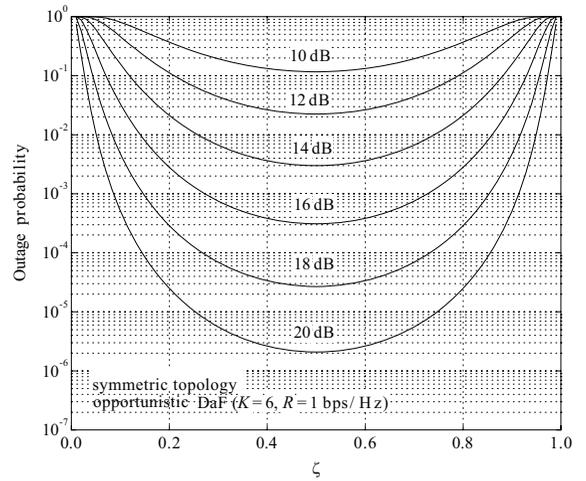


Fig. 5. Outage probability as a function of power allocation ζ for opportunistic DaF relaying at the end-to-end spectral efficiency $R = 1$ bps/Hz in symmetric channels when $\mathcal{P}_{\text{tot}}/N_0 = 10, 12, 14, 16, 18,$ and 20 dB. $K = 6$ and $\Omega_{S_k} = \Omega_{kD} = 1$, $k = 1, 2, \dots, 6$.

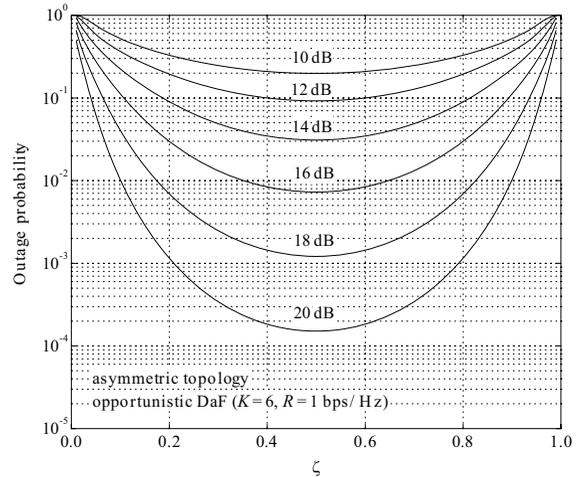


Fig. 6. Outage probability as a function of power allocation ζ for opportunistic DaF relaying at the end-to-end spectral efficiency $R = 1$ bps/Hz in asymmetric channels when $\mathcal{P}_{\text{tot}}/N_0 = 10, 12, 14, 16, 18,$ and 20 dB. $K = 6$ and $\{\Omega_{S_k}\}_{k=1}^K = \{\Omega_{kD}\}_{k=1}^K = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$.

compare the performance of (i) opportunistic AaF relaying, (ii) AaF relaying with equal-power MR transmissions, and (iii) AaF relaying with single-relay selection based on the maximum average channel gain $\max_{k \in \mathcal{S}_{\text{relay}}} \Omega_{S_k}$ (or $\max_{k \in \mathcal{S}_{\text{relay}}} \Omega_{kD}$). Note that for the symmetric case, single-relay selection based on the average channel gains amounts to selecting just one relay randomly (since all relays have the same average channel gains towards the source and destination) and transmitting with full relaying power $\mathcal{P}_{\text{relay}}$. For the asymmetric case studied, single-relay selection based on the average channel gains amounts to selecting the relay with $\Omega_{S_k} = \Omega_{kD} = 4.5$.

Both figures reveal significant gains of opportunistic AaF relaying compared to MR transmission with AaF strategy. This is true even for the symmetric scenario, where all relays have the same average channel gains. For AaF schemes, the mutual information in (24) involves the vectorial addition of multiple relay terms. This is due to the fact that MR transmitted signals

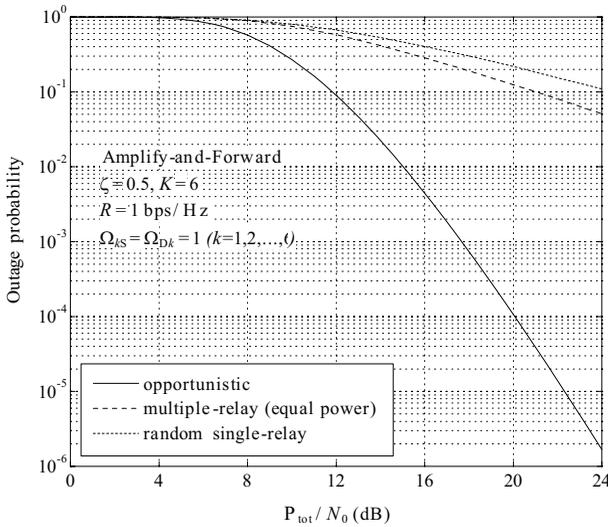


Fig. 7. Outage probability as a function of $\mathcal{P}_{\text{tot}}/N_0$ for the AaF strategy at the end-to-end spectral efficiency $R = 1$ bps/Hz in symmetric channels. $\zeta = 0.5$, $K = 6$, and $\Omega_{S_k} = \Omega_{kD} = 1$, $k = 1, 2, \dots, 6$. Opportunistic relaying is compared with equal-power MR transmissions and random single-relay selection.

do not necessarily add constructively at the receiver, and therefore, their superposition does not increase linearly with the number of relays. Both figures show again that selecting a relay based on average channel gain removes the potential benefits of opportunistic relaying.

Finally, Figs. 9 and 10 show the performance of opportunistic AaF relaying as a function of ζ for the symmetric and asymmetric scenarios, respectively. It is shown that $\zeta = 0.5$ is not the optimal ζ^* , even for the symmetric scenario. This is due to the fact that relays also amplify the noise introduced in the first reception and therefore, it is important at the final destination to overcome noise introduced at the relay *as well as* at the destination. From that perspective, it is preferable to allocate more power at the opportunistic relay than to the source. The figures also show that choice of $\zeta = 0.5$ (in the absence of network CSI) incurs a small performance loss of a few parts-per-million of the outage probability. Therefore, $\zeta = 0.5$ is a reasonable choice in practice, since it gives a near-optimal performance.

VI. CONCLUSION

This paper presented *opportunistic* relaying protocols and analyzed outage performance under an aggregate power constraint. In particular, we proposed simple opportunistic relaying protocols that can be implemented in distributed manners without requiring global CSI. We showed that both reactive and proactive opportunistic DaF relaying are outage-optimal. We further showed that opportunistic AaF relaying significantly outperforms AaF strategies based on equal-power MR transmissions, when only local CSI is available. Additionally, opportunistic AaF relaying is outage-optimal among single relay selection schemes. Finally, we demonstrated that equal power allocation between the source and opportunistic relay gives a near-optimal performance without requiring global CSI.

Proactive opportunistic relaying allows all relays, except a single opportunistic relay, to enter an idle mode even during

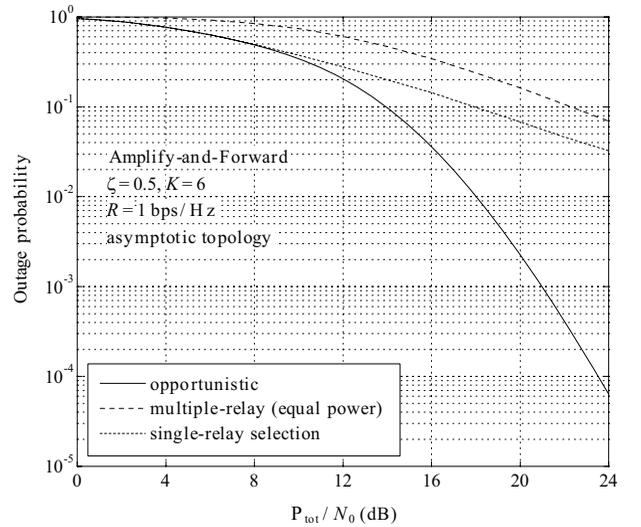


Fig. 8. Outage probability as a function of $\mathcal{P}_{\text{tot}}/N_0$ for the AaF strategy at the end-to-end spectral efficiency $R = 1$ bps/Hz in asymmetric channels. $\zeta = 0.5$, $K = 6$, and $\{\Omega_{S_k}\}_{k=1}^K = \{\Omega_{kD}\}_{k=1}^K = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$. Opportunistic relaying is compared with equal-power MR transmissions and single-relay selection based on average channel gains.

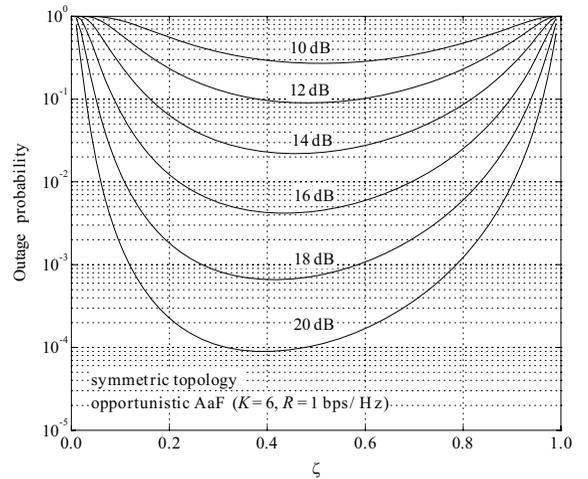


Fig. 9. Outage probability as a function of power allocation ζ for opportunistic AaF relaying at the end-to-end spectral efficiency $R = 1$ bps/Hz in symmetric channels when $\mathcal{P}_{\text{tot}}/N_0 = 10, 12, 14, 16, 18, \text{ and } 20$ dB. $K = 6$ and $\Omega_{S_k} = \Omega_{kD} = 1$, $k = 1, 2, \dots, 6$. The optimal value of ζ is not equal to $\zeta^* = 0.5$ for all values of $\mathcal{P}_{\text{tot}}/N_0$.

the source transmission, which reduces the reception energy cost in the network. Therefore, our proactive strategy can be viewed as energy-efficient routing in the network. In contrast, the reactive schemes require all relays to receive information during the source transmission and consequently, scale the reception energy proportionally with the network size. This overhead may not be negligible, especially in battery-operated terminals.

Our results reveal that relays in cooperative communications can be viewed not only as active re-transmitters, but also as distributed sensors of the wireless channel. Cooperative relays can be useful even when they do not transmit, provided that they cooperatively *listen*. In that way, cooperation benefits can be cultivated with simple radio implementation.

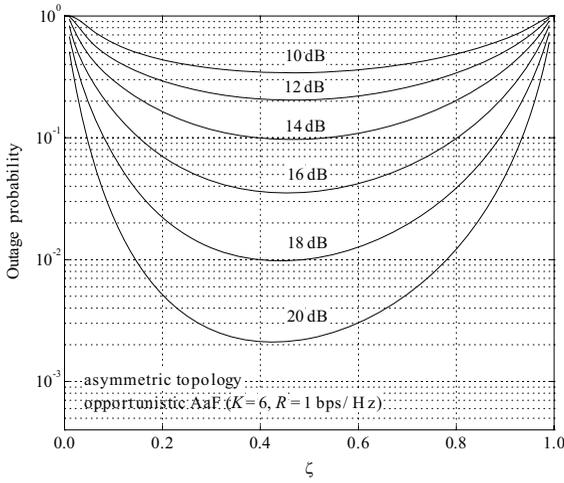


Fig. 10. Outage probability as a function of power allocation ζ for opportunistic AaF relaying at the end-to-end spectral efficiency $R = 1$ bps/Hz in asymmetric channels when $\mathcal{P}_{\text{tot}}/N_0 = 10, 12, 14, 16, 18,$ and 20 dB. $K = 6$ and $\{\Omega_{S_k}\}_{k=1}^K = \{\Omega_{kD}\}_{k=1}^K = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$. The optimal value of ζ is not equal to $\zeta^* = 0.5$ for all values of $\mathcal{P}_{\text{tot}}/N_0$.

APPENDIX

SOME DISTRIBUTIONS INVOLVING EXPONENTIAL VARIATES

A. A Sum Distribution

Theorem 2 (Sum Distribution): Let $Y_n \sim \Upsilon(1/\mu_n)$, $n = 1, 2, \dots, N$, be N statistically independent and not necessarily identically distributed (i.n.i.d.) exponential r.v.'s. Then, the p.d.f. of a sum $X = \sum_{n=1}^N Y_n$ is given by

$$p_X(x) = \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\tau_i(\mathbf{A})} \mathcal{X}_{i,j}(\mathbf{A}) \frac{\mu_{(i)}^{-j}}{(j-1)!} x^{j-1} e^{-x/\mu_{(i)}}, \quad x \geq 0 \quad (29)$$

where $\mathbf{A} = \text{diag}(\mu_1, \mu_2, \dots, \mu_N)$, $\varrho(\mathbf{A})$ is the number of distinct diagonal elements of \mathbf{A} , $\mu_{(1)} > \mu_{(2)} > \dots > \mu_{(\varrho(\mathbf{A}))}$ are the distinct diagonal elements in decreasing order, $\tau_i(\mathbf{A})$ is the multiplicity of $\mu_{(i)}$, and $\mathcal{X}_{i,j}(\mathbf{A})$ is the (i, j) th characteristic coefficient of \mathbf{A} [23].¹³ The cumulative distribution function (c.d.f.) of X is given by

$$F_X(x) = 1 - \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\tau_i(\mathbf{A})} \sum_{k=0}^{j-1} \frac{\mathcal{X}_{i,j}(\mathbf{A})}{k!} \left(\frac{x}{\mu_{(i)}}\right)^k e^{-x/\mu_{(i)}}, \quad x \geq 0. \quad (30)$$

Proof: Since Y_1, Y_2, \dots, Y_N are statistically independent, the characteristic function (c.f.) of X is

$$\Phi_X(j\omega) \triangleq \mathbb{E}\{e^{j\omega X}\} = \prod_{n=1}^N (1 - j\omega\mu_n)^{-1}. \quad (31)$$

Using a partial fraction decomposition of (31) with the

¹³Let Σ be an $n \times n$ Hermitian matrix with the eigenvalues $\sigma_1, \sigma_2, \dots, \sigma_n$ in any order, $\varrho(\Sigma)$ be the number of distinct eigenvalues, $\sigma_{(1)} > \sigma_{(2)} > \dots > \sigma_{(\varrho(\Sigma))}$ be the distinct eigenvalues in decreasing order, and $\tau_i(\Sigma)$ be the multiplicity of $\sigma_{(i)}$. Then, the (i, j) th characteristic coefficient $\mathcal{X}_{i,j}(\Sigma)$, $i = 1, 2, \dots, \varrho(\Sigma)$, $j = 1, 2, \dots, \tau_i(\Sigma)$, is defined as a partial fraction expansion coefficient of $\det(\mathbf{I}_n + \xi\Sigma)^{-1}$ such that

$$\det(\mathbf{I}_n + \xi\Sigma)^{-1} = \prod_{i=1}^{\varrho(\Sigma)} (1 + \xi\sigma_{(i)})^{-\tau_i(\Sigma)}$$

characteristic coefficients, we obtain the p.d.f. of X as

$$\begin{aligned} p_X(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega x} \Phi_X(j\omega) d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega x} \det(\mathbf{I}_N + j\omega\mathbf{A})^{-1} d\omega \\ &= \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\tau_i(\mathbf{A})} \frac{\mathcal{X}_{i,j}(\mathbf{A})}{2\pi} \int_{-\infty}^{\infty} e^{j\omega x} (1 + j\omega\mu_{(i)})^{-j} d\omega \\ &= \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\tau_i(\mathbf{A})} \mathcal{X}_{i,j}(\mathbf{A}) \frac{\mu_{(i)}^{-j}}{(j-1)!} x^{j-1} e^{-x/\mu_{(i)}} u(x) \end{aligned} \quad (32)$$

where \mathbf{I}_N denotes the $N \times N$ identity matrix and $u(x)$ is the Heaviside step function.

From (32), we obtain the c.d.f. of X as

$$\begin{aligned} F_X(x) &= \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\tau_i(\mathbf{A})} \mathcal{X}_{i,j}(\mathbf{A}) \frac{\mu_{(i)}^{-j}}{(j-1)!} \int_0^x t^{j-1} e^{-t/\mu_{(i)}} dt \\ &= \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\tau_i(\mathbf{A})} \mathcal{X}_{i,j}(\mathbf{A}) \left\{ 1 - \frac{1}{(j-1)!} \Gamma\left(j, \frac{x}{\mu_{(i)}}\right) \right\} \\ &= 1 - \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\tau_i(\mathbf{A})} \frac{\mathcal{X}_{i,j}(\mathbf{A})}{(j-1)!} \Gamma\left(j, \frac{x}{\mu_{(i)}}\right) \end{aligned} \quad (33)$$

where the last equality follows from the fact that the sum of all the characteristic coefficients is equal to one [23], and $\Gamma(n, z)$ is the incomplete gamma function defined by

$$\Gamma(n, z) \triangleq \int_z^{\infty} t^{n-1} e^{-t} dt. \quad (34)$$

Finally, using the identity [26, eq. (8.352.2)]

$$\Gamma(n, z) = (n-1)! e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{k!}, \quad n \text{ positive integer} \quad (35)$$

yields the desired result (30). \square

The following corollaries are two extreme cases of Theorem 2, i.e., the cases of all distinct μ_n 's and all equal μ_n 's.

Corollary 1: If all of μ_n 's are distinct ($\varrho(\mathbf{A}) = N$ and $\tau_i(\mathbf{A}) = 1$ in Theorem 2), then we have

$$\begin{aligned} p_X(x) &= \sum_{i=1}^N \left\{ \prod_{\substack{j=1 \\ j \neq i}}^N \left(1 - \frac{\mu_j}{\mu_i}\right)^{-1} \right\} \frac{e^{-x/\mu_i}}{\mu_i}, \quad x \geq 0 \quad (36) \\ &= \sum_{i=1}^{\varrho(\Sigma)} \sum_{j=1}^{\tau_i(\Sigma)} \mathcal{X}_{i,j}(\Sigma) (1 + \xi\sigma_{(i)})^{-j} \end{aligned}$$

where ξ is a scalar constant such that $\mathbf{I}_n + \xi\Sigma$ is nonsingular and $\mathcal{X}_{i,j}(\Sigma)$ can be determined by

$$\begin{aligned} \mathcal{X}_{i,j}(\Sigma) &= \frac{1}{\varpi_{i,j}! \sigma_{(i)}^{\varpi_{i,j}}} \\ &\times \left[\frac{d^{\varpi_{i,j}}}{dv^{\varpi_{i,j}}} (1 + v\sigma_{(i)})^{\tau_i(\Sigma)} \det(\mathbf{I}_n + v\Sigma)^{-1} \right] \Big|_{v=-\frac{1}{\sigma_{(i)}}} \end{aligned}$$

with $\varpi_{i,j} = \tau_i(\Sigma) - j$.

and

$$F_X(x) = 1 - \sum_{i=1}^N \left\{ \prod_{\substack{j=1 \\ j \neq i}}^N \left(1 - \frac{\mu_j}{\mu_i}\right)^{-1} \right\} e^{-x/\mu_i}, \quad x \geq 0. \quad (37)$$

Proof: When all of μ_n 's are distinct, the characteristic coefficients of \mathcal{A} become [23]

$$\mathcal{X}_{i,1}(\mathcal{A}) = \prod_{\substack{j=1 \\ j \neq i}}^N \left(1 - \frac{\mu_j}{\mu_i}\right)^{-1}, \quad i = 1, 2, \dots, N. \quad (38)$$

The proof follows immediately from Theorem 2 and (38). \square

Corollary 2: If $\mu_n = \mu$, $n = 1, 2, \dots, N$ ($\varrho(\mathcal{A}) = 1$ and $\tau_1(\mathcal{A}) = N$ in Theorem 2), then we have

$$p_X(x) = \frac{\mu^{-N}}{(N-1)!} x^{N-1} e^{-x/\mu}, \quad x \geq 0 \quad (39)$$

and

$$F_X(x) = 1 - \sum_{k=0}^{N-1} \frac{1}{k!} \left(\frac{x}{\mu}\right)^k e^{-x/\mu}, \quad x \geq 0. \quad (40)$$

Proof: When all of μ_n 's are equal, the characteristic coefficients of \mathcal{A} become [23]

$$\mathcal{X}_{1,j}(\mathcal{A}) = \begin{cases} 0, & j = 1, 2, \dots, N-1 \\ 1, & j = N. \end{cases} \quad (41)$$

The proof follows immediately from Theorem 2 and (41). \square

We remark that Corollary 2 agrees with the well-known fact that a sum of N i.i.d. exponential r.v.'s has a central chi-squared distribution with $2N$ degrees of freedom.

B. A Product-Ratio Distribution

Theorem 3 (Product-Ratio Distribution): Let

$$Y_1 \sim \Upsilon(1/\mu_1)$$

$$Y_2 \sim \Upsilon(1/\mu_2)$$

be statistically i.n.i.d. exponential r.v.'s. Suppose a product-ratio X of the form

$$X = \frac{Y_1 Y_2}{a\mu_1 + Y_2}, \quad a > 0. \quad (42)$$

Then, we have

$$p_X(x) = \frac{1}{\mu_1 \mu_2} \int_0^\infty \frac{a\mu_1 + z}{z} \exp\left\{-\frac{x(a\mu_1 + z)}{\mu_1 z} - \frac{z}{\mu_2}\right\} dz \quad (43)$$

and

$$F_X(x) = 1 - \frac{1}{\mu_2} \int_0^\infty \exp\left\{-\frac{x(a\mu_1 + z)}{\mu_1 z} - \frac{z}{\mu_2}\right\} dz \quad (44)$$

where $x \geq 0$.

Proof: Note that

$$\begin{aligned} F_X(x) &= \Pr\left\{\frac{Y_1 Y_2}{a\mu_1 + Y_2} \leq x\right\} \\ &= \mathbb{E}_{Y_2}\{F_{X|Y_2}(x)\} \\ &= \mathbb{E}_{Y_2}\left\{1 - \exp\left[-\frac{x(a\mu_1 + Y_2)}{\mu_1 Y_2}\right]\right\} \\ &= 1 - \frac{1}{\mu_2} \int_0^\infty \exp\left\{-\frac{x(a\mu_1 + z)}{\mu_1 z} - \frac{z}{\mu_2}\right\} dz \quad (45) \end{aligned}$$

as desired. The p.d.f. of X in (43) follows immediately from differentiating (44) with respect to x . \square

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