A Unified Spectral Analysis of Generalized Time-Hopping Spread-Spectrum Signals in the Presence of Timing Jitter

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Invited Paper

***Abstract*—This paper characterizes the power spectral density (PSD) of various time-hopping spread-spectrum (TH-SS) signaling schemes in the presence of random timing jitter, which is characterized typically by a discrete-time stationary random process (independent of the TH sequences and data sequence) with known statistical properties. A flexible model for a general TH-SS signal is proposed and a unified spectral analysis of this generalized TH-SS signal is carried out using a systematic and tractable technique. The key idea is to express the basic baseband pulse in terms of its Fourier transform which allows flexibility in specifying different TH formats throughout the general derivation. The power spectrum of various TH-SS signaling schemes can then be obtained as a special case of the generalized PSD results. Although general PSD results are first obtained for arbitrary timing jitter statistics, specific results are then given for the cases of practical interest, namely, uniform and Gaussian distributed jitter. Applications of this unified spectral analysis includes: 1) clocked TH by a random sequence; 2) framed TH by a random sequence; and 3) framed TH by a pseudorandom periodic sequence. Detailed descriptions of these different TH techniques will be given where the first two techniques employ a random sequence (stochastic model) and the third technique employs a pseudorandom sequence (deterministic model).**

***Index Terms*—Power spectral density (PSD), spectral analysis, spread-spectrum, time-hopping, timing jitter, ultra-wide bandwidth (UWB).**

I. Introduction

RECENTLY, there has been an interest in ultra-wide bandwidth (UWB) time-hopping spread-spectrum (TH-SS) multiple-access techniques for both commercial and military applications [1]–[13]. The UWB TH-SS radio communicates with pulses of short duration, thereby spreading the energy of the radio signal very thinly over several GHz [1]–[4]. The techniques for generating UWB signals have been around for more than three decades [14]. A description of early work in this area can be found in [15].

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The key motivations for using UWB TH-SS radio are the ability to highly resolve multipath, as well as the availability of the technology to implement and generate UWB signals with relatively low complexity. The fine delay resolution properties make UWB TH-SS radio a viable candidate for communications in dense multipath environments such as short-range or indoor wireless communications [16]–[19]. UWB TH-SS radios, operating with low transmission power in an extremely large transmission bandwidth, have also been under consideration for future military networks because they inherently provide a covertness property with low probability of detection and interception (LPD/LPI) capabilities [5]–[7]. Anti-jam capability of UWB TH-SS signals based on the signaling format proposed in [1]–[4] have been analyzed [11], [12]. Recent efforts in reducing the receiver complexity [20]–[27] make us envision a wide diffusion of the UWB-radio technology.

Since gigahertz bandwidth allocations are not available at the proposed frequencies for commercial applications, these radios must operate under Part 15 Federal Communication Committee (FCC) regulations and are treated as spurious interference to all other systems [28]. In addition, UWB TH-SS radios operating over the highly populated frequency range, must contend with a variety of interfering signals and must also insure that they do not interfere with narrowband radio as well as navigation systems operating in these dedicated bands. For military UWB TH-SS radios, the major objective is to minimize the probability of detection and interception by the enemy. This requires transmitting TH-SS signals with extremely low spectral content. On the other hand, they must maintain the average power level required for reliable communications. All of this implies that the spectral occupancy and composition of a chosen TH-SS signaling technique play an important role in the design of UWB TH-SS systems. For example, the ability of the receiver to reject narrowband interference, the ability of the transmitter to avoid interfering with other radio systems, and the probability of detection/interception by a nonintended receiver can be estimated from the power spectral density (PSD) of the time-hopped pulse trains.

Evaluation of the PSD for *ideal synchronous data pulse streams* based upon *stochastic theory* is well documented in the literature [29]–[33]. Here, ideal synchronous data pulse streams refers to a baseband data waveform with a fixed transmission interval and fixed transmission epochs where the underlying data sequence has known statistical properties and the transmitted waveform is a single known pulse shape. A time-series approach to spectral analysis, using *nonstochastic theory* based upon time averages, provides an alternative technique to analyzing the spectral properties of digital signals [34]. The duality between the two complementary theories, *stochastic theory using ensemble averages* and *nonstochastic theory using time averages*, can be found in [35], where the "fraction-of-time distribution" concept is introduced as a means of obtaining a probabilistic interpretation of time averages as expected values or ensemble averages.

Evaluation of the PSD for *nonideal* data pulse streams, due for instance to implementation imperfections, is also of interest. One such example is data asymmetry [36] where the rising and falling transition time instants are offset by fixed amounts relative to the nominal ones. A complete PSD analysis for this form of nonideal condition was documented in [37] and [38]. Another source of imperfection is timing jitter [39] where the amount of timing shift per transmission interval is random and is typically characterized by a discrete-time stationary random process with known statistical properties [40]. General expressions for the PSD of *M*-ary digital pulse streams in the presence of timing jitter were derived using *stochastic theory* in [41]. The empirical spectral analysis of a time-jittered pulse-amplitude modulated signal was carried out using the nonstochastic (time-series) approach in [34].

Previous work in the literature for evaluation of the PSD of SS signals has focused mainly on *ideal* direct-sequence SS signals [42]–[46]. The PSD of *ideal* frequency-hopped SS signals was derived in [43]. However, the effect of timing jitter was not included in these analyses. The purpose of this paper is to characterize the PSD of various TH-SS signaling schemes in the presence of random timing jitter, using a stochastic approach similar to [29]–[32], [41]. A flexible model for general TH-SS signals is proposed and a unified spectral analysis of this generalized TH-SS signal (given in (1) of Section II) is carried out using a systematic and tractable technique. The key idea is to express the basic baseband pulse in terms of its Fourier transform. This formulation allows flexibility in specifying different TH formats throughout the general derivation. The power spectrum of a variety of TH-SS signaling schemes can then be obtained as a special case of the generalized PSD results. Although general PSD results are first obtained for arbitrary timing jitter statistics, specific results are then given for the cases of practical interest, namely, uniform and Gaussian distributed jitter. Applications of this unified spectral analysis includes: 1) clocked TH by a random sequence; 2) framed TH by a random sequence; and 3) framed TH by a pseudorandom periodic sequence. Detailed description of these different TH techniques will be given in Section V; however, it should be noted in passing that the first two techniques employ a random TH sequence (stochastic model), and the third technique employs a pseudorandom TH sequence (deterministic model). A stochastic model is of particular interest in applications such as random signal detection and/or interception, whereas a deterministic model is of interest in multiple-access applications.

II. SIGNAL MODEL AND PRELIMINARY DEFINITIONS

An equivalent baseband model of a generalized TH-SS signal in the presence of timing jitter can be written as

$$s(t) = \sum_{n=-\infty}^{\infty} a_n w(t - nT_1 - b_n T_2 - c_n T_3 - \epsilon_n) \tag{1}$$

where $w(t)$ is a baseband pulse waveform, and $\{a_n\}$ and $\{b_n\}$ are arbitrary stochastic (randomly chosen) sequences. The sequence $\{c_n\}$ is a deterministic periodic sequence with period $N_{\mathrm{p}}^{\mathrm{c}}$. The sequence $\{\epsilon_n\}$ is a discrete-time stationary random process representing timing jitter [40]. It is assumed that $\{a_n\}$, $\{b_n\}$, and $\{\epsilon_n\}$ are stationary and mutually independent of each other.

The key idea of this unified analysis is to express the basic baseband pulse waveform $w(t)$ as

$$w(t) = \int_{-\infty}^{\infty} W(f) e^{+j2\pi f t} df \tag{2}$$

where $W(f)$ is the Fourier transform of $w(t)$. In this formulation, the TH-SS signal $s(t)$ becomes (3), shown at the bottom of the page. The main advantage of this formulation can be readily seen from (3) in that it allows one to write the different TH sequences (whether stochastic or deterministic) as well as timing jitter as a product of exponentials (phase modulations). Note also that it detaches the derivation from a specific shape of the pulse waveform $w(t)$.

It is convenient to define a new sequence $\beta_n(y)$ as

$$\beta_n(y) \triangleq a_n e^{+j2\pi y b_n T_2} e^{+j2\pi y c_n T_3} e^{+j2\pi y \epsilon_n}. \tag{4}$$

The mean of $\beta_n(y)$ is given by

$$\overline{\beta}_n(y) = \mathbb{E}\{\beta_n(y)\} = \mathbb{E}\left\{a_n e^{+j2\pi y b_n T_2} e^{+j2\pi y c_n T_3} e^{+j2\pi y \epsilon_n}\right\} \tag{5}$$

and the cross-covariance function of the two sequences $\beta_n(y)$ and $\beta_m(z)$ is given by

$$\begin{aligned} &K_\beta(n; m-n, y, z) \\ &\quad = \mathbb{E}\left\{\left[\beta_n(y) - \overline{\beta}_n(y)\right]\left[\beta_m(z) - \overline{\beta}_m(z)\right]^*\right\} \qquad (6) \\ &\quad = \Big[\mathbb{E}\left\{a_n a_m^* e^{+j2\pi y(b_n T_2 + \epsilon_n)} e^{-j2\pi z(b_m T_2 + \epsilon_m)}\right\} \\ &\qquad - \mathbb{E}\left\{a_n e^{+j2\pi y(b_n T_2 + \epsilon_n)}\right\} \mathbb{E}\left\{a_m^* e^{-j2\pi z(b_m T_2 + \epsilon_m)}\right\}\Big] \\ &\qquad \times e^{+j2\pi y c_n T_3} e^{-j2\pi z c_m T_3}. \qquad (7) \end{aligned}$$

$$s(t) = \sum_{n=-\infty}^{\infty} a_n \int_y W(y) e^{-j2\pi y b_n T_2} e^{-j2\pi y c_n T_3} e^{-j2\pi y \epsilon_n} e^{-j2\pi y n T_1} e^{+j2\pi y t} dy. \tag{3}$$

Since the sequences $\{a_n\}$, $\{b_n\}$, and $\{\epsilon_n\}$ are mutually independent, the mean and cross-covariance become

$$\overline{\beta}_n(y) = \mathbb{E}\left\{a_n\right\} \Phi_{b_n}(yT_2)\Phi_{\epsilon_n}(y)e^{+j2\pi y c_n T_3} \tag{8}$$

and

$$\begin{aligned} K_\beta(n; m-n, y, z) &= \Big[R_a(n; m-n)\Phi_{b_n,b_m}(yT_2, -zT_2)\Phi_{\epsilon_n,\epsilon_m}(y, -z) \\ &\quad - \overline{a}_n\overline{a}_m^* \Phi_{b_n}(yT_2)\Phi_{\epsilon_n}(y)\Phi_{b_m}(-zT_2)\Phi_{\epsilon_m}(-z)\Big] \\ &\quad \times e^{+j2\pi y c_n T_3} e^{-j2\pi z c_m T_3} \end{aligned} \tag{9}$$

respectively, where

$$\Phi_{b_n,b_m}(y,z) = \mathbb{E}\left\{e^{j2\pi(yb_n+zb_m)}\right\}$$
$$\Phi_{\epsilon_n,\epsilon_m}(y,z) = \mathbb{E}\left\{e^{j2\pi(y\epsilon_n+z\epsilon_m)}\right\}$$
$$\Phi_{b_n}(y) = \mathbb{E}\left\{e^{j2\pi y b_n}\right\}$$

and

$$\Phi_{\epsilon_n}(y) = \mathbb{E}\left\{e^{j2\pi y \epsilon_n}\right\}.$$

The functions $\overline{a}_n$ and $R_a(n;l)$ are the mean and correlation function of the sequence $\{a_n\}$ given by

$$\overline{a}_n \triangleq \mathbb{E}\left\{a_n\right\} \tag{10}$$

and

$$R_a(n;l) \triangleq \mathbb{E}\left\{a_n a_{n+l}^*\right\}. \tag{11}$$

Letting $l = m - n$, it can be shown that, for the stationary stochastic sequences $\{a_n\}$, $\{b_n\}$, and $\{\epsilon_n\}$, and deterministic periodic sequence $\{c_n\}$ with period N_p^c, $K_\beta(n; l, y, z)$ is periodic in n with period N_p^c.[1] Using this periodic property, the covariance function of $s(t)$ is derived in Appendix I as shown in (12), at the bottom of the page, where $\overline{s}(t)$ is the mean of $s(t)$ and $\delta_\mathrm{D}(\cdot)$ is the Dirac delta function [49].

The product of $\overline{s}(t)$ and its time-displaced version $\overline{s}^*(t+\tau)$ is also of interest in calculating the PSD of TH-SS signals, and is given by (13), shown at the bottom of the page. For deterministic periodic sequence $\{c_n\}$ with period N_p^c, $\overline{\beta}_n(y)$ in (5) is periodic in n with period N_p^c. Using this periodic property, the product in (13) is calculated in Appendix II as shown in (14), at the bottom of the page.

III. PSD of Generalized TH-SS Signals With Arbitrary Jitter

In general, the PSD $S_s(f)$ of the generalized TH-SS signal $s(t)$ consists of continuous, as well as discrete components [29], [30], namely $S_s^\mathrm{c}(f)$ and $S_s^\mathrm{d}(f)$, respectively.

Irrespective of the properties of the sequences $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, and $\{\epsilon_n\}$, the generalized TH-SS signal $s(t)$ is itself wide-sense cyclostationary since $K_s(t;\tau)$ is, in addition to being a function of τ, a periodic function of t. Therefore, the continuous spectrum of $s(t)$ is

$$S_s^\mathrm{c}(f) = \mathcal{F}_\tau\left\{\langle K_s(t;\tau)\rangle_t\right\} \tag{15}$$

[1]The kind of stationarity conditions required for the sequence $\{a_n\}$, $\{b_n\}$, and $\{\epsilon_n\}$, to obtain $K_\beta(n, m-n, y, z)$ periodic in n can be relaxed. In fact, sufficient conditions are that the sequence $\{a_n\}$ is wide-sense stationary and the sequences $\{b_n\}$, and $\{\epsilon_n\}$ are second-order stationary in the *strict sense*. Definitions for different kinds of stationarity can be found in [35], [47], and [48].

$$\begin{aligned} K_s(t;\tau) &= \mathbb{E}\left\{\left[s(t)-\overline{s}(t)\right]\left[s(t+\tau)-\overline{s}(t+\tau)\right]^*\right\} \\ &= \sum_{l=-\infty}^{\infty}\int_y\int_z W(y)W^*(z)e^{+j2\pi z l T_1}\sum_{k=-\infty}^{\infty}\delta_\mathrm{D}\left(y-z-\frac{k}{N_\mathrm{p}^\mathrm{c}T_1}\right) \\ &\quad \times \frac{1}{N_\mathrm{p}^\mathrm{c}T_1}\sum_{n=0}^{N_\mathrm{p}^\mathrm{c}-1}K_\beta(n;l,-y,-z)e^{-j2\pi(y-z)nT_1}e^{+j2\pi(y-z)t}e^{-j2\pi z\tau}\,dydz \end{aligned} \tag{12}$$

$$\overline{s}(t)\overline{s}^*(t+\tau) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\mathbb{E}\left\{a_n w(t-nT_1-b_nT_2-c_nT_3-\epsilon_n)\right\}\mathbb{E}\left\{a_m^* w^*(t+\tau-mT_1-b_mT_2-c_mT_3-\epsilon_m)\right\} \tag{13}$$

$$\begin{aligned} \overline{s}(t)\overline{s}^*(t+\tau) &= \frac{1}{N_\mathrm{p}^\mathrm{c}T_1}\sum_{k=-\infty}^{\infty}W\left(\frac{k}{N_\mathrm{p}^\mathrm{c}T_1}\right)\sum_{n=0}^{N_\mathrm{p}^\mathrm{c}-1}\overline{\beta}_n\left(-\frac{k}{N_\mathrm{p}^\mathrm{c}T_1}\right)e^{-j2\pi(k/N_\mathrm{p}^\mathrm{c})n} \\ &\quad \times \frac{1}{N_\mathrm{p}^\mathrm{c}T_1}\sum_{l=-\infty}^{\infty}W^*\left(-\frac{l}{N_\mathrm{p}^\mathrm{c}T_1}\right)\sum_{m=0}^{N_\mathrm{p}^\mathrm{c}-1}\overline{\beta}_m^*\left(\frac{l}{N_\mathrm{p}^\mathrm{c}T_1}\right)e^{-j2\pi(l/N_\mathrm{p}^\mathrm{c})m} \\ &\quad \times \exp\left\{+j2\pi\left(\frac{k+l}{N_\mathrm{p}^\mathrm{c}T_1}\right)t\right\}\exp\left\{+j2\pi\left(\frac{l}{N_\mathrm{p}^\mathrm{c}T_1}\right)\tau\right\} \end{aligned} \tag{14}$$

where $\langle\cdot\rangle_t$ denotes time average, and $\mathcal{F}_\tau\{\cdot\}$ denotes the Fourier transform operator with respect to the variable τ. For the stationary stochastic sequences $\{a_n\}, \{b_n\}, \{\epsilon_n\}$, and deterministic periodic sequence $\{c_n\}$ with period N_p^c, $S_s^\mathrm{c}(f)$ is calculated in Appendix III as

$$S_s^\mathrm{c}(f) = S_w(f) S_\beta(f) \tag{16}$$

where

$$S_w(f) = \frac{1}{T_1} |W(f)|^2 \tag{17}$$

is the (normalized) energy spectral density of the pulse shape $w(t)$ and

$$S_\beta(f) = \sum_{l=-\infty}^{\infty} \left[\frac{1}{N_\mathrm{p}^\mathrm{c}} \sum_{n=0}^{N_\mathrm{p}^\mathrm{c}-1} K_\beta(n; l, f, f) \right] e^{-j2\pi f l T_1}. \tag{18}$$

The function $K_\beta(n; l, f, f)$ is the covariance function of the new sequence $\beta_n(f)$ and is given by

$$\begin{aligned} K_\beta(n; l, f, f) &= \mathbb{E}\left\{ \left[\beta_n(f) - \overline{\beta}_n(f)\right] \left[\beta_{n+l}(f) - \overline{\beta}_{n+l}(f)\right]^* \right\} \quad (19)\\ &= \Big[R_a(n;l) \Phi_{b_n, b_{n+l}}(fT_2, -fT_2) \Phi_{\epsilon_n, \epsilon_{n+l}}(f, -f) \\ &\quad - \overline{a}_n \overline{a}_{n+l}^* \Phi_{b_n}(fT_2) \Phi_{\epsilon_n}(f) \Phi_{b_{n+l}}(-fT_2) \Phi_{\epsilon_{n+l}}(-f) \Big] \\ &\quad \times e^{+j2\pi f (c_n - c_{n+l}) T_3}. \quad (20) \end{aligned}$$

The function $S_\beta(f)$ can be interpreted conveniently as the continuous spectrum of the "*equivalent* stationary sequence," that is, the periodicity (with period N_p^c) of the "covariance" function due to the periodic behavior of the deterministic $\{c_n\}$ is averaged out before the discrete Fourier transform operation.

The discrete PSD is found from

$$S_s^\mathrm{d}(f) = \mathcal{F}_\tau \left\{ \langle \overline{s}(t) \overline{s}^*(t+\tau) \rangle_t \right\}. \tag{21}$$

For the stationary stochastic sequences $\{a_n\}$, $\{b_n\}$, $\{\epsilon_n\}$, and deterministic periodic sequence $\{c_n\}$ with period N_p^c, $S_s^\mathrm{d}(f)$ is calculated in Appendix IV as

$$\begin{aligned} S_s^\mathrm{d}(f) &= \frac{1}{(N_\mathrm{p}^\mathrm{c} T_1)^2} \\ &\times \sum_{l=-\infty}^{\infty} \left| W\left(\frac{l}{N_\mathrm{p}^\mathrm{c} T_1}\right) \right|^2 \left| G_\beta\left(\frac{l}{N_\mathrm{p}^\mathrm{c} T_1}\right) \right|^2 \delta_\mathrm{D}\left(f - \frac{l}{N_\mathrm{p}^\mathrm{c} T_1}\right) \end{aligned} \tag{22}$$

where

$$G_\beta(f) = \sum_{n=0}^{N_\mathrm{p}^\mathrm{c}-1} \overline{\beta}_n(f) e^{j2\pi f n T_1}. \tag{23}$$

IV. PSD of Generalized TH-SS Signals With Specific Timing Jitter Statistics

The results obtained in Section III are general and apply to arbitrary stationary timing jitter statistics. In this section, the expressions for the PSD are evaluated for two specific cases of practical interest, namely uniform distributed timing jitter (UDTJ) and Gaussian distributed timing jitter (GDTJ).

A. *Uniform Distributed Timing Jitter*

In this section, the PSD of the generalized TH-SS signal in the presence of UDTJ is considered. Specifically, $\{\epsilon_n\}$ is modeled as a sequence of independent identically distributed (i.i.d.) uniform random variables with probability density function (pdf)

$$f_{\epsilon_n}(x) = \begin{cases} \frac{1}{\Delta}, & \Delta_1 < x < \Delta_2 \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

where $\Delta = \Delta_2 - \Delta_1$. Note that, in general, ϵ_n has nonzero mean and, thus, this model includes the *asymmetry* of the timing jitter.

It is easy to show that

$$\Phi_{\epsilon_n}(f) = \left[\frac{\sin(\pi f \Delta)}{(\pi f \Delta)}\right] e^{+j2\pi f((\Delta_2+\Delta_1)/2)} \tag{25}$$

and

$$\Phi_{\epsilon_n, \epsilon_{n+l}}(f, -f) = \begin{cases} 1, & l = 0 \\ \left[\frac{\sin(\pi f \Delta)}{(\pi f \Delta)}\right]^2, & l \neq 0. \end{cases} \tag{26}$$

Therefore, from (20) see (27), shown at the bottom of the page and from (8)

$$\begin{aligned} \overline{\beta}_n(f) &= \overline{a}_n \Phi_{b_n}(fT_2) e^{+j2\pi f c_n T_3} \\ &\quad \times \left[\frac{\sin(\pi f \Delta)}{(\pi f \Delta)}\right] e^{+j2\pi f((\Delta_2+\Delta_1)/2)}. \end{aligned} \tag{28}$$

Finally, substituting (27) and (28) into (18) and (23), respectively, and using (16), (17), and (22) of Section III, the continuous and discrete PSD components of a generalized TH-SS signal in the presence of i.i.d. UDTJ (asymmetric or symmetric) over the interval Δ becomes (29) and (30), shown at the bottom of the next page, respectively. Note that (29) and (30) only depend on the interval, Δ, of the timing jitter and, thus, are independent of the jitter asymmetry.

B. *Gaussian Distributed Timing Jitter*

In this section, the PSD of the generalized TH-SS signal in the presence of GDTJ is considered. Specifically, $\{\epsilon_n\}$ is modeled as a sequence of i.i.d. Gaussian random variables with pdf

$$f_{\epsilon_n}(x) = \frac{1}{\sqrt{2\pi}\Delta} e^{-(x-\theta)^2/(2\Delta^2)}. \tag{31}$$

In general, $\theta \neq 0$ and it is important to point out again that this model includes the *asymmetry* of the timing jitter. It is easy to verify that

$$\Phi_{\epsilon_n}(f) = e^{+\left\{j(2\pi f\theta) - (1/2)(2\pi f \Delta)^2\right\}} \tag{32}$$

$$K_\beta(n; l, f, f) = \begin{cases} R_a(n;0) - |\overline{a}_n|^2 |\Phi_{b_n}(fT_2)|^2 \left[\frac{\sin(\pi f \Delta)}{(\pi f \Delta)}\right]^2, & l = 0 \\ \left[R_a(n;l) \Phi_{b_n, b_{n+l}}(fT_2, -fT_2) - \overline{a}_n \overline{a}_{n+l}^* |\Phi_{b_n}(fT_2)|^2\right] e^{+j2\pi f(c_n - c_{n+l})T_3} \left[\frac{\sin(\pi f \Delta)}{(\pi f \Delta)}\right]^2, & l \neq 0 \end{cases} \tag{27}$$

and

$$\Phi_{\epsilon_n,\epsilon_{n+l}}(f,-f)=\begin{cases}1, & l=0\\ e^{-(2\pi f\Delta)^2}, & l\neq 0.\end{cases} \tag{33}$$

Therefore, from (20), see (34), shown at the bottom of the page and from (8)

$$\overline{\beta}_n(f)=\overline{a}_n\Phi_{b_n}(fT_2)e^{+j2\pi fc_nT_3}e^{+\{j(2\pi f\theta)-(1/2)(2\pi f\Delta)^2\}}. \tag{35}$$

Finally, substituting (34) and (35) into (18) and (23), respectively, and using (16), (17), and (22) of Section III, give the desired results as shown in (36) and (37), shown at the bottom of the next page, respectively. Equations (36) and (37) are the continuous and discrete PSD components of a generalized TH-SS signal in the presence of i.i.d. GDTJ with mean θ and standard deviation Δ. Note that, as for the UDTJ case, (36) and (37) are independent of θ and, thus, are independent of the jitter asymmetry.

C. Limiting Cases

The limiting case of $\Delta_1\rightarrow\Delta_2$ or $\Delta\rightarrow 0$ for both UDTJ and GDTJ implies constant clock delay with no timing jitter. Therefore, the PSD expressions of an ideal synchronous generalized TH-SS signal can be obtained from (29) and (30) (for UDTJ), or (36) and (37) (for GDTJ) by setting $\Delta_1\rightarrow\Delta_2$ or $\Delta\rightarrow 0$.

V. Applications

A. Clocked TH by a Random Sequence

Consider a digitally controlled TH-SS signal which produces random transmissions at multiples of the basic clock period T_c. This signaling technique, known as clocked TH by a random sequence (CTHRS), can be modeled as

$$s_{\mathrm{CTHRS}}(t)=\sum_{n=-\infty}^{\infty}a_nw(t-nT_{\mathrm{c}}-\epsilon_n) \tag{38}$$

where $\{a_n\}$ is an i.i.d. random sequence. Typically, $\{a_n\}$ is an (unbalanced or balanced) binary sequence with

$$\Pr\{a_n\}=\begin{cases}p, & a_n=1\\ 1-p, & a_n=0.\end{cases} \tag{39}$$

Note that $s_{\mathrm{CTHRS}}(t)$ can be obtained from the generalized TH-SS signal $s(t)$ of (1) by setting

$$\begin{aligned}&T_1=T_{\mathrm{c}}\\ &T_2=0 \quad \text{or} \quad b_n=0\\ &T_3=0 \quad \text{or} \quad c_n=0.\end{aligned}$$

It can be shown that $\{a_n\}$ has correlation function

$$R_a(l)=\begin{cases}p, & l=0\\ p^2, & l\neq 0\end{cases} \tag{40}$$

and mean $\overline{a}=p$.

$$\begin{aligned}S_s^{\mathrm{c}}(f)=&\frac{1}{T_1}|W(f)|^2\left\{\frac{1}{N_{\mathrm{p}}^{\mathrm{c}}}\sum_{n=0}^{N_{\mathrm{p}}^{\mathrm{c}}-1}\left[R_a(n;0)-|\overline{a}_n|^2|\Phi_{b_n}(fT_2)|^2\left[\frac{\sin(\pi f\Delta)}{(\pi f\Delta)}\right]^2\right]\right\}\\ &+\frac{1}{T_1}|W(f)|^2\left\{\sum_{l\neq 0}\left[\frac{1}{N_{\mathrm{p}}^{\mathrm{c}}}\sum_{n=0}^{N_{\mathrm{p}}^{\mathrm{c}}-1}\left[R_a(n;l)\Phi_{b_n,b_{n+l}}(fT_2,-fT_2)-\overline{a}_n\overline{a}_{n+l}^*|\Phi_{b_n}(fT_2)|^2\right]e^{+j2\pi f(c_n-c_{n+l})T_3}\right]e^{-j2\pi flT_1}\right\}\\ &\times\left[\frac{\sin(\pi f\Delta)}{(\pi f\Delta)}\right]^2\end{aligned} \tag{29}$$

$$\begin{aligned}S_s^{\mathrm{d}}(f)=&\frac{1}{(N_{\mathrm{p}}^{\mathrm{c}}T_1)^2}\sum_{l=-\infty}^{\infty}\left|W\left(\frac{l}{N_{\mathrm{p}}^{\mathrm{c}}T_1}\right)\right|^2\left|\sum_{n=0}^{N_{\mathrm{p}}^{\mathrm{c}}-1}\overline{a}_n\Phi_{b_n}\left(\frac{lT_2}{N_{\mathrm{p}}^{\mathrm{c}}T_1}\right)e^{+j2\pi(lT_3/(N_{\mathrm{p}}^{\mathrm{c}}T_1))c_n}e^{+j2\pi(ln/N_{\mathrm{p}}^{\mathrm{c}})}\right|^2\\ &\times\left[\frac{\sin(\pi\frac{l}{N_{\mathrm{p}}^{\mathrm{c}}T_1}\Delta)}{(\pi\frac{l\Delta}{N_{\mathrm{p}}^{\mathrm{c}}T_1})}\right]^2\delta_{\mathrm{D}}\left(f-\frac{l}{N_{\mathrm{p}}^{\mathrm{c}}T_1}\right)\end{aligned} \tag{30}$$

$$K_\beta(n;l,f,f)=\begin{cases}R_a(n;0)-|\overline{a}_n|^2|\Phi_{b_n}(fT_2)|^2e^{-(2\pi f\Delta)^2}, & l=0\\ \left[R_a(n;l)\Phi_{b_n,b_{n+l}}(fT_2,-fT_2)-\overline{a}_n\overline{a}_{n+l}^*|\Phi_{b_n}(fT_2)|^2\right] & \\ \quad\times e^{+j2\pi f(c_n-c_{n+l})T_3}e^{-(2\pi f\Delta)^2}, & l\neq 0\end{cases} \tag{34}$$

For an independent stationary stochastic sequence $\{\epsilon_n\}$, it can be shown using (20) and (8) that

$$K_\beta(n;l,f,f) = \begin{cases} p - p^2 \left|\Phi_{\epsilon_n}(f)\right|^2, & l = 0 \\ 0, & l \neq 0 \end{cases} \quad (41)$$

and

$$\overline{\beta}_n(f) = p\, \Phi_{\epsilon_n}(f). \quad (42)$$

Since $\{\epsilon_n\}$ is stationary, $K_\beta(n;l,f,f)$ and $\overline{\beta}_n(f)$ are independent of n, or equivalently the periods of $K_\beta(n;\cdot,\cdot,\cdot)$ and $\overline{\beta}_n(\cdot)$ are one.

Finally, substituting (41) and (42) into (18) and (23), respectively, and using (16), (17), and (22) of Section III, gives the desired results

$$S^{\mathrm{c}}_{\mathrm{CTHRS}}(f) = \frac{1}{T_{\mathrm{c}}} \left|W(f)\right|^2 \left\{p - p^2 \left|\Phi_{\epsilon_n}(f)\right|^2\right\} \quad (43)$$

and

$$S^{\mathrm{d}}_{\mathrm{CTHRS}}(f) = \frac{p^2}{T_{\mathrm{c}}^2} \times \sum_{l=-\infty}^{\infty} \left|W\left(\frac{l}{T_{\mathrm{c}}}\right)\right|^2 \left|\Phi_{\epsilon_n}\left(\frac{l}{T_{\mathrm{c}}}\right)\right|^2 \delta_{\mathrm{D}}\left(f - \frac{l}{T_{\mathrm{c}}}\right). \quad (44)$$

Equations (43) and (44) are the continuous and discrete PSD components of a CTHRS signal in the presence of arbitrary stationary timing jitter statistics. It can be seen that a CTHRS removes a portion of the signal's power, namely the amount given by $(p^2/T_{\mathrm{c}}) \int_{-\infty}^{\infty} |W(f)|^2 \left|\Phi_{\epsilon_n}(f)\right|^2 df$, and converts it into the set of spectral lines (indicated by the delta functions) which occur at multiples of the clock frequency $1/T_{\mathrm{c}}$.

The results for UDTJ can now be obtained easily by substituting (25) into (43) and (44) as

$$S^{\mathrm{c}}_{\mathrm{CTHRS}}(f) = \frac{1}{T_{\mathrm{c}}} \left|W(f)\right|^2 \left\{p - p^2 \left[\frac{\sin(\pi f \Delta)}{(\pi f \Delta)}\right]^2\right\} \quad (45)$$

and

$$S^{\mathrm{d}}_{\mathrm{CTHRS}}(f) = \frac{p^2}{T_{\mathrm{c}}^2} \times \sum_{l=-\infty}^{\infty} \left|W\left(\frac{l}{T_{\mathrm{c}}}\right)\right|^2 \left[\frac{\sin(\pi \frac{l}{T_{\mathrm{c}}} \Delta)}{(\pi \frac{l\Delta}{T_{\mathrm{c}}})}\right]^2 \delta_{\mathrm{D}}\left(f - \frac{l}{T_{\mathrm{c}}}\right). \quad (46)$$

The results for GDTJ can also be obtained similarly by substituting (32) into (43) and (44) as

$$S^{\mathrm{c}}_{\mathrm{CTHRS}}(f) = \frac{1}{T_{\mathrm{c}}} \left|W(f)\right|^2 \left\{p - p^2 e^{-(2\pi f \Delta)^2}\right\} \quad (47)$$

and

$$S^{\mathrm{d}}_{\mathrm{CTHRS}}(f) = \frac{p^2}{T_{\mathrm{c}}^2} \times \sum_{l=-\infty}^{\infty} \left|W\left(\frac{l}{T_{\mathrm{c}}}\right)\right|^2 e^{-(2\pi(l\Delta/T_{\mathrm{c}}))^2} \delta_{\mathrm{D}}\left(f - \frac{l}{T_{\mathrm{c}}}\right). \quad (48)$$

Note that, both the UDTJ and GDTJ models include the *asymmetry* of the timing jitter; however, the PSD results for CTHRS signals are independent of the jitter asymmetry as was the case for the PSD results for the generalized TH-SS signals derived in Section IV.

As pointed out in Section IV-C, the PSD results of an ideal CTHRS signal (in the absence of timing jitter) can be obtained by setting $\Delta_1 \to \Delta_2$ or $\Delta \to 0$ as

$$S^{\mathrm{c}}_{\mathrm{CTHRS}}(f) = \frac{1}{T_{\mathrm{c}}} \left|W(f)\right|^2 \left\{p - p^2\right\} \quad (49)$$

and

$$S^{\mathrm{d}}_{\mathrm{CTHRS}}(f) = \frac{p^2}{T_{\mathrm{c}}^2} \sum_{l=-\infty}^{\infty} \left|W\left(\frac{l}{T_{\mathrm{c}}}\right)\right|^2 \delta_{\mathrm{D}}\left(f - \frac{l}{T_{\mathrm{c}}}\right). \quad (50)$$

As a check, the results in (49) and (50) agree with those in [29, p. 64, eq. (2.59)] by letting $S_1(f) = W(f)$ and $S_2(f) = 0$ in the latter. Consider a purely periodic transmission with one pulse every T_{c} seconds ($p = 1$), i.e., a dc-biased square wave. It

$$S^{\mathrm{c}}_s(f) = \frac{1}{T_1} \left|W(f)\right|^2 \left\{\frac{1}{N^{\mathrm{c}}_{\mathrm{p}}} \sum_{n=0}^{N^{\mathrm{c}}_{\mathrm{p}}-1} \left[R_a(n;0) - |\overline{a}_n|^2 |\Phi_{b_n}(fT_2)|^2 e^{-(2\pi f\Delta)^2}\right]\right\}$$
$$+ \frac{1}{T_1} \left|W(f)\right|^2 \left\{\sum_{l\neq 0} \left[\frac{1}{N^{\mathrm{c}}_{\mathrm{p}}} \sum_{n=0}^{N^{\mathrm{c}}_{\mathrm{p}}-1} \left[R_a(n;l)\Phi_{b_n,b_{n+l}}(fT_2,-fT_2) - \overline{a}_n \overline{a}^*_{n+l} |\Phi_{b_n}(fT_2)|^2\right] e^{+j2\pi f(c_n - c_{n+l})T_3}\right] e^{-j2\pi f l T_1}\right\}$$
$$\times e^{-(2\pi f\Delta)^2} \quad (36)$$

$$S^{\mathrm{d}}_s(f) = \frac{1}{(N^{\mathrm{c}}_{\mathrm{p}} T_1)^2} \sum_{l=-\infty}^{\infty} \left|W\left(\frac{l}{N^{\mathrm{c}}_{\mathrm{p}} T_1}\right)\right|^2 \left|\sum_{n=0}^{N^{\mathrm{c}}_{\mathrm{p}}-1} \overline{a}_n \Phi_{b_n}\left(\frac{lT_2}{N^{\mathrm{c}}_{\mathrm{p}} T_1}\right) e^{+j2\pi\left(lT_3/(N^{\mathrm{c}}_{\mathrm{p}} T_1)\right)c_n} e^{+j2\pi(ln/N^{\mathrm{c}}_{\mathrm{p}})}\right|^2$$
$$\times e^{-\left(2\pi l\Delta/(N^{\mathrm{c}}_{\mathrm{p}} T_1)\right)^2} \delta_{\mathrm{D}}\left(f - \frac{l}{N^{\mathrm{c}}_{\mathrm{p}} T_1}\right) \quad (37)$$

can be seen from (49) and (50) that, when the timing jitter is absent, the continuous spectrum disappears and the spectrum contains only spectral lines spaced at $1/T_c$ as expected. When the timing jitter is present, however, the continuous spectrum still exists in (43). Therefore, timing jitter "helps" from the viewpoint of smoothing the spectrum.

B. Framed TH by a Random Sequence

Some TH systems may require somewhat regular spacing between pulses, in addition to clocked time locations. This signaling technique is referred to as framed TH by a random sequence (FTHRS). The FTHRS signal $s_{\rm FTHRS}(t)$ can be written as

$$s_{\rm FTHRS}(t) = \sum_{n=-\infty}^{\infty} w(t - nT_f - b_n T_c - \epsilon_n) \tag{51}$$

where T_f is a frame time or average pulse repetition time, and $\{b_n\}$ is an integer-valued i.i.d. random TH sequence with

$$\Pr\{b_n = m\} = \begin{cases} p_m, & 0 \le m < N_h \\ 0, & \text{otherwise .} \end{cases} \tag{52}$$

The values of the sequence elements are in the range

$$0 \le b_n < N_h \tag{53}$$

such that

$$N_h T_c \le T_f. \tag{54}$$

As indicated above, the number N_h of possible hop times can be any number from 1 (a regular pulse train with pulses spaced T_f apart), to T_f/T_c (one pulse at any randomly selected clock time within each frame of T_f seconds), the latter resembling pulse position modulation (PPM). The signal $s_{\rm FTHRS}(t)$ can be obtained from the generalized TH-SS signal $s(t)$ of (1) by setting

$$\begin{aligned} a_n &= 1 \\ T_1 &= T_f \\ T_2 &= T_c \\ T_3 &= 0 \qquad \text{or} \qquad c_n = 0. \end{aligned}$$

It is easy to verify that

$$\Phi_{b_n}(f) = \sum_{m=0}^{N_h-1} p_m e^{+j(2\pi f m)} \tag{55}$$

and

$$\Phi_{b_n, b_{n+l}}(f, -f) = \begin{cases} 1, & l = 0 \\ \left| \sum_{m=0}^{N_h-1} p_m e^{+j(2\pi f m)} \right|^2, & l \ne 0. \end{cases} \tag{56}$$

For independent stationary stochastic sequences $\{b_n\}$ and $\{\epsilon_n\}$, it can be shown using (20) and (8) that

$$K_\beta(n; l, f, f) = \begin{cases} 1 - \left| \sum_{m=0}^{N_h-1} p_m e^{+j(2\pi f m T_c)} \right|^2 |\Phi_{\epsilon_n}(f)|^2, & l = 0 \\ 0, & l \ne 0 \end{cases} \tag{57}$$

and

$$\overline{\beta}_n(f) = \sum_{m=0}^{N_h-1} p_m e^{+j(2\pi f m T_c)} \Phi_{\epsilon_n}(f). \tag{58}$$

Since the sequences $\{b_n\}$ and $\{\epsilon_n\}$ are stationary, $K_\beta(n; l, f, f)$ and $\overline{\beta}_n(f)$ are independent of n, or equivalently the periods of $K_\beta(n; \cdot, \cdot, \cdot)$ and $\overline{\beta}_n(\cdot)$ are one.

Substituting (57) and (58) into (18) and (23), respectively, and using (16), (17), and (22) of Section III, the continuous and discrete PSD components of a FTHRS signal in the presence of arbitrary stationary timing jitter statistics become

$$S^c_{\rm FTHRS}(f) = \frac{1}{T_f} |W(f)|^2 \times \left\{ 1 - \left| \sum_{m=0}^{N_h-1} p_m e^{+j(2\pi f m T_c)} \right|^2 |\Phi_{\epsilon_n}(f)|^2 \right\} \tag{59}$$

and (60), shown at the bottom of the page. Note that FTHRS, characterized by $\{p_m\}$, removes the power from the continuous spectrum by the amount

$$(1/T_f) \int_{-\infty}^{\infty} |W(f)|^2 \left| \sum_{m=0}^{N_h-1} p_m e^{+j(2\pi f m T_c)} \right|^2 |\Phi_{\epsilon_n}(f)|^2 \, df.$$

This, however, reappears in the discrete spectrum as expected. Roughly speaking, the framing condition causes "energy transfer" from the continuous PSD to spectral lines ($1/T_f$ apart) in the discrete PSD.

The results for UDTJ can now be obtained by substituting (25) into (59) and (60) as

$$S^c_{\rm FTHRS}(f) = \frac{1}{T_f} |W(f)|^2 \times \left\{ 1 - \left| \sum_{m=0}^{N_h-1} p_m e^{+j(2\pi f m T_c)} \right|^2 \left[\frac{\sin(\pi f \Delta)}{(\pi f \Delta)} \right]^2 \right\} \tag{61}$$

and (62), shown at the bottom of the next page. The results for GDTJ can also be obtained similarly by substituting (32) into (59) and (60) as

$$S^c_{\rm FTHRS}(f) = \frac{1}{T_f} |W(f)|^2 \times \left\{ 1 - \left| \sum_{m=0}^{N_h-1} p_m e^{+j(2\pi f m T_c)} \right|^2 e^{-(2\pi f \Delta)^2} \right\} \tag{63}$$

$$S^d_{\rm FTHRS}(f) = \frac{1}{T_f^2} \sum_{l=-\infty}^{\infty} \left| W\left(\frac{l}{T_f}\right) \right|^2 \left| \sum_{m=0}^{N_h-1} p_m e^{+j\left(2\pi (l/T_f) m T_c\right)} \right|^2 \left| \Phi_{\epsilon_n}\left(\frac{l}{T_f}\right) \right|^2 \delta_D\left(f - \frac{l}{T_f}\right) \tag{60}$$

and (64), shown at the bottom of the page.

The PSD results of an ideal FTHRS signal (in the absence of timing jitter) become

$$S^{c}_{\mathrm{FTHRS}}(f) = \frac{1}{T_{\mathrm{f}}} |W(f)|^2 \left\{ 1 - \left| \sum_{m=0}^{N_{\mathrm{h}}-1} p_m e^{+j(2\pi f m T_{\mathrm{c}})} \right|^2 \right\} \tag{65}$$

and (66), shown at the bottom of the page.

C. *Framed TH by a Pseudorandom Periodic Sequence*

For multiple-access applications, the TH sequence is provided to transmitter and receiver with a previously agreed upon sequence. This can be best described by a deterministic model known as framed TH by a pseudorandom periodic sequence (FTHPPS). This signal $s_{\mathrm{FTHPPS}}(t)$ can be written as

$$s_{\mathrm{FTHPPS}}(t) = \sum_{n=-\infty}^{\infty} w(t - nT_{\mathrm{f}} - c_n T_{\mathrm{c}} - \epsilon_n) \tag{67}$$

where $\{c_n\}$ is a pseudorandomly generated periodic sequence, with integer values in the range

$$0 \le c_n < N_{\mathrm{h}}. \tag{68}$$

The period of this pseudorandom sequence is denoted by N^{c}_{p} and, therefore, the period T^{c}_{p} of the transmitted signal is $T^{c}_{\mathrm{p}} = N^{c}_{\mathrm{p}} T_{\mathrm{f}}$. Note that the selection of the transmission time in each frame is deterministic.

The signal $s_{\mathrm{FTHPPS}}(t)$ can be obtained from the generalized TH-SS signal $s(t)$ of (1) by setting

$$\begin{aligned} a_n &= 1 \\ T_1 &= T_{\mathrm{f}} \\ T_2 &= 0 \qquad \text{or} \qquad b_n = 0 \\ T_3 &= T_{\mathrm{c}}. \end{aligned}$$

Therefore, from (20)

$$K_{\beta}(n; l, f, f) = \begin{cases} 1 - |\Phi_{\epsilon_n}(f)|^2, & l = 0 \\ 0, & l \neq 0 \end{cases} \tag{69}$$

and from (8)

$$\overline{\beta}_n(f) = e^{+j2\pi f c_n T_{\mathrm{c}}} \Phi_{\epsilon_n}(f). \tag{70}$$

Note that $K_{\beta}(n; l, f, f)$ is independent of n, but $\overline{\beta}_n(\cdot)$ is periodic with period N^{c}_{p}.

For arbitrary timing statistics, the PSD components becomes

$$S^{c}_{\mathrm{FTHPPS}}(f) = \frac{1}{T_{\mathrm{f}}} |W(f)|^2 \left\{ 1 - |\Phi_{\epsilon_n}(f)|^2 \right\} \tag{71}$$

and (72), shown at the bottom of the page. In the case of UDTJ, simple substitution shows that

$$S^{c}_{\mathrm{FTHPPS}}(f) = \frac{1}{T_{\mathrm{f}}} |W(f)|^2 \left\{ 1 - \left[\frac{\sin(\pi f \Delta)}{(\pi f \Delta)} \right]^2 \right\} \tag{73}$$

and (74), shown at the bottom of the next page.

$$S^{d}_{\mathrm{FTHRS}}(f) = \frac{1}{T_{\mathrm{f}}^2} \sum_{l=-\infty}^{\infty} \left| W\left(\frac{l}{T_{\mathrm{f}}}\right) \right|^2 \left| \sum_{m=0}^{N_{\mathrm{h}}-1} p_m e^{+j(2\pi (l/T_{\mathrm{f}}) m T_{\mathrm{c}})} \right|^2 \left[\frac{\sin(\pi \frac{l}{T_{\mathrm{f}}} \Delta)}{(\pi \frac{l\Delta}{T_{\mathrm{f}}})} \right]^2 \delta_{\mathrm{D}}\left(f - \frac{l}{T_{\mathrm{f}}}\right) \tag{62}$$

$$S^{d}_{\mathrm{FTHRS}}(f) = \frac{1}{T_{\mathrm{f}}^2} \sum_{l=-\infty}^{\infty} \left| W\left(\frac{l}{T_{\mathrm{f}}}\right) \right|^2 \left| \sum_{m=0}^{N_{\mathrm{h}}-1} p_m e^{+j(2\pi (l/T_{\mathrm{f}}) m T_{\mathrm{c}})} \right|^2 e^{-(2\pi (l\Delta/T_{\mathrm{f}}))^2} \delta_{\mathrm{D}}\left(f - \frac{l}{T_{\mathrm{f}}}\right) \tag{64}$$

$$S^{d}_{\mathrm{FTHRS}}(f) = \frac{1}{T_{\mathrm{f}}^2} \sum_{l=-\infty}^{\infty} \left| W\left(\frac{l}{T_{\mathrm{f}}}\right) \right|^2 \left| \sum_{m=0}^{N_{\mathrm{h}}-1} p_m e^{+j(2\pi (l/T_{\mathrm{f}}) m T_{\mathrm{c}})} \right|^2 \delta_{\mathrm{D}}\left(f - \frac{l}{T_{\mathrm{f}}}\right) \tag{66}$$

$$S^{d}_{\mathrm{FTHPPS}}(f) = \frac{1}{(N^{c}_{\mathrm{p}} T_{\mathrm{f}})^2} \sum_{l=-\infty}^{\infty} \left| W\left(\frac{l}{N^{c}_{\mathrm{p}} T_{\mathrm{f}}}\right) \right|^2 \left| \sum_{n=0}^{N^{c}_{\mathrm{p}}-1} e^{+j2\pi (l c_n T_{\mathrm{c}}/(N^{c}_{\mathrm{p}} T_{\mathrm{f}}))} e^{+j2\pi (ln/N^{c}_{\mathrm{p}})} \right|^2 \left| \Phi_{\epsilon_n}\left(\frac{l}{N^{c}_{\mathrm{p}} T_{\mathrm{f}}}\right) \right|^2 \delta_{\mathrm{D}}\left(f - \frac{l}{N^{c}_{p} T_{\mathrm{f}}}\right) \tag{72}$$

In the case of GDTJ

$$S^{\mathrm{c}}_{\mathrm{FTHPPS}}(f) = \frac{1}{T_{\mathrm{f}}}\,|W(f)|^2\left\{1 - e^{-(2\pi f\Delta)^2}\right\} \tag{75}$$

and (76), shown at the bottom of the page. In the absence of timing jitter, the PSD components become

$$S^{\mathrm{c}}_{\mathrm{FTHPPS}}(f) = 0 \tag{77}$$

and (78), shown at the bottom of the page. Note again that timing jitter "helps" in smoothing the spectrum.

VI. Comments on TH Sequence Design

Typical sequence designs are based on the PSD of an ideal FTHPPS signal, which can be rewritten as

$$S_{\mathrm{FTHPPS}}(f) = \frac{1}{(N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}})^2}|W(f)|^2 \times \underbrace{\left|\sum_{n=0}^{N^{\mathrm{c}}_{\mathrm{p}}-1} e^{+j2\pi f(nT_{\mathrm{f}}+c_nT_{\mathrm{c}})}\right|^2}_{\triangleq C(f)} \sum_{k=-\infty}^{\infty}\delta_{\mathrm{D}}\left(f - \frac{k}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}}\right). \tag{79}$$

Note that the delta functions which compose the line spectral density are separated by the reciprocal of one period $1/(N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}})$ of the pseudorandomly time-hopped signal. For large $N^{\mathrm{c}}_{\mathrm{p}}$ these spectral line spacing are small providing an opportunity to spread the power more evenly across the transmission bandwidth and/or to minimize the amount of power residing in any single spectral line. The addition of nontrivial data modulation or the presence of timing jitter on the signal will further smooth this line spectral density as a function of frequency.

The envelope of the lines in the spectral density has two frequency-dependent factors, namely $|W(f)|^2$ and $C(f)$, the latter being TH sequence dependent. Note that when T_{f} is an integer multiple of T_{c}, $C(f)$ is periodic in f with period $1/T_{\mathrm{c}}$. This implies that, attempts to influence (or shape) one portion of the frequency spectrum by sequence design will have an effect on another portion of the spectrum. There may be an opportunity to make $C(f)$ better than approximately flat as a function of frequency, e.g., make $C(f) \approx 1/|W(f)|^2$ over a specified interval.

There may be some lines in the PSD which cannot be reduced by clever design of TH sequences. For example, suppose that $T_{\mathrm{f}}/T_{\mathrm{c}} = m'/n'$, where m' and n' are relatively prime integers. Then, $C(f) = (N^{\mathrm{c}}_{\mathrm{p}})^2$ for all frequencies f that are integer multiples of n'/T_{c}, and lines exist in $S_{\mathrm{FTHPPS}}(f)$ at these frequencies. The heights of these spectral lines are independent of the TH sequence and can only be influenced by the energy spectrum $|W(f)|^2$ of the basic baseband pulse waveform.

VII. Conclusion

General expressions for the PSD of a variety of TH-SS signaling schemes in the presence of arbitrary timing jitter are derived using *stochastic theory*. A flexible model for a general TH-SS signal is proposed and a unified spectral analysis of this generalized TH-SS signal is carried out using a systematic and tractable technique. The power spectrum of a variety of TH-SS signaling schemes can then be obtained as a special case of the generalized PSD results. Although general PSD results are first obtained for arbitrary timing jitter statistics, specific results are then given for the cases of practical interest, namely, uniform and Gaussian distributed jitter. The results show that the *asymmetry* of the timing jitter does *not* affect the PSD, and that timing jitter "helps" in smoothing the line spectrum. Applications of this unified spectral analysis includes: 1) clocked TH

$$S^{\mathrm{d}}_{\mathrm{FTHPPS}}(f) = \frac{1}{(N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}})^2}\sum_{l=-\infty}^{\infty}\left|W\left(\frac{l}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}}\right)\right|^2\left|\sum_{n=0}^{N^{\mathrm{c}}_{\mathrm{p}}-1} e^{+j2\pi(lc_nT_{\mathrm{c}}/(N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}))}e^{+j2\pi(ln/N^{\mathrm{c}}_{\mathrm{p}})}\right|^2 \times \left[\frac{\sin(\pi\frac{l\Delta}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}})}{(\pi\frac{l\Delta}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}})}\right]^2\delta_{\mathrm{D}}\left(f - \frac{l}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}}\right) \tag{74}$$

$$S^{\mathrm{d}}_{\mathrm{FTHPPS}}(f) = \frac{1}{(N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}})^2}\sum_{l=-\infty}^{\infty}\left|W\left(\frac{l}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}}\right)\right|^2\left|\sum_{n=0}^{N^{\mathrm{c}}_{\mathrm{p}}-1} e^{+j2\pi(lc_nT_{\mathrm{c}}/(N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}))}e^{+j2\pi(ln/N^{\mathrm{c}}_{\mathrm{p}})}\right|^2 \times e^{-\left(2\pi(l\Delta/N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}})\right)^2}\delta_{\mathrm{D}}\left(f - \frac{l}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}}\right) \tag{76}$$

$$S^{\mathrm{d}}_{\mathrm{FTHPPS}}(f) = \frac{1}{(N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}})^2}\sum_{l=-\infty}^{\infty}\left|W\left(\frac{l}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}}\right)\right|^2\left|\sum_{n=0}^{N^{\mathrm{c}}_{\mathrm{p}}-1} e^{+j2\pi(lc_nT_{\mathrm{c}}/(N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}))}e^{+j2\pi(ln/N^{\mathrm{c}}_{\mathrm{p}})}\right|^2\delta_{\mathrm{D}}\left(f - \frac{l}{N^{\mathrm{c}}_{\mathrm{p}}T_{\mathrm{f}}}\right) \tag{78}$$

by a random sequence; 2) framed TH by a random sequence; and 3) framed TH by a pseudorandom periodic sequence. The PSD of an ideal synchronous TH-SS signal is shown to be a special case of the results obtained in this paper, and can be obtained as a limiting case when the timing jitter goes to zero (i.e., $\Delta_1 \to \Delta_2$ or $\Delta \to 0$).

APPENDIX I
COVARIANCE FUNCTION OF THE GENERALIZED TH-SS SIGNAL $s(t)$

The covariance function of the generalized TH-SS signal $s(t)$, is given by (80), shown at the bottom of the page. In terms of the Fourier transform of $w(t)$, (80) becomes the second equation shown at the bottom of the page.

The covariance function $K_s(t;\tau)$ can be rewritten as (81), shown at the bottom of the page. Since $K_\beta(n;l,y,z)$ is periodic in n with period $N_{\rm p}^{\rm c}$, it is easy to show that

$$\sum_{n=-\infty}^{\infty} K_\beta(n;l,-y,-z)e^{-j2\pi(y-z)nT_1} = \sum_{n=0}^{N_{\rm p}^{\rm c}-1} K_\beta(n;l,-y,-z)e^{-j2\pi(y-z)nT_1} \sum_{i=-\infty}^{\infty} e^{-j2\pi(y-z)iN_{\rm p}^{\rm c}T_1}. \tag{82}$$

Using the Poisson sum formula

$$\sum_{n=-\infty}^{\infty} e^{-j2\pi xnT} = \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta_{\rm D}\left(x-\frac{k}{T}\right) \tag{83}$$

and substituting (82) in (81) gives (84), shown at the bottom of the page.

APPENDIX II
EVALUATION OF THE PRODUCT $\overline{s}(t)\overline{s}^*(t+\tau)$

The product $\overline{s}(t)\overline{s}^*(t+\tau)$ is evaluated as shown in the first equation at the bottom of the next page.

In terms of the definition given in (5), $\overline{s}(t)\overline{s}^*(t+\tau)$ becomes

$$\overline{s}(t)\overline{s}^*(t+\tau) = \sum_{n=-\infty}^{\infty}\int_y W(y)\overline{\beta}_n(-y)\, e^{-j2\pi ynT_1}e^{+j2\pi yt}dy \times \sum_{m=-\infty}^{\infty}\int_z W^*(z)\overline{\beta}_m^*(-z)\, e^{+j2\pi zmT_1}e^{-j2\pi zt}e^{-j2\pi z\tau}dz. \tag{85}$$

$$\begin{aligned} K_s(t;\tau) &= \mathbb{E}\left\{[s(t)-\overline{s}(t)]\,[s(t+\tau)-\overline{s}(t+\tau)]^*\right\} \\ &= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\mathbb{E}\left\{a_n a_m^* w(t-nT_1-b_nT_2-c_nT_3-\epsilon_n)w^*(t+\tau-mT_1-b_mT_2-c_mT_3-\epsilon_m)\right\} \\ &\quad - \mathbb{E}\left\{a_n w(t-nT_1-b_nT_2-c_nT_3-\epsilon_n)\right\}\mathbb{E}\left\{a_m^* w^*(t+\tau-mT_1-b_mT_2-c_mT_3-\epsilon_m)\right\} \end{aligned} \tag{80}$$

$$\begin{aligned} K_s(t;\tau) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\int_y\int_z &\Big[\mathbb{E}\Big\{a_n a_m^* e^{-j2\pi y(b_nT_2+\epsilon_n)}e^{+j2\pi z(b_mT_2+\epsilon_m)}\Big\} - \mathbb{E}\Big\{a_n e^{-j2\pi y(b_nT_2+\epsilon_n)}\Big\}\mathbb{E}\Big\{a_m^* e^{+j2\pi z(b_mT_2+\epsilon_m)}\Big\}\Big] \\ &\times e^{-j2\pi yc_nT_3}e^{+j2\pi zc_mT_3}W(y)W^*(z)e^{-j2\pi ynT_1}e^{+j2\pi zmT_1}e^{+j2\pi(y-z)t}e^{-j2\pi z\tau}dydz \end{aligned}$$

$$K_s(t;\tau) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\int_y\int_z K_\beta(n;m-n,-y,-z)W(y)W^*(z)e^{-j2\pi ynT_1}e^{+j2\pi zmT_1}e^{+j2\pi(y-z)t}e^{-j2\pi z\tau}dydz \tag{81}$$

$$\begin{aligned} K_s(t;\tau) = &\sum_{l=-\infty}^{\infty}\int_y\int_z W(y)W^*(z)e^{+j2\pi zlT_1}\sum_{k=-\infty}^{\infty}\delta_{\rm D}\left(y-z-\frac{k}{N_{\rm p}^{\rm c}T_1}\right) \\ &\times\frac{1}{N_{\rm p}^{\rm c}T_1}\sum_{n=0}^{N_{\rm p}^{\rm c}-1}K_\beta(n;l,-y,-z)e^{-j2\pi(y-z)nT_1}e^{+j2\pi(y-z)t}e^{-j2\pi z\tau}dydz \end{aligned} \tag{84}$$

Since $\overline{\beta}_n(y)$ is periodic in n with period $N_{\rm p}^{\rm c}$. Therefore

$$\begin{aligned}&\sum_{n=-\infty}^{\infty}\overline{\beta}_n(-y)e^{-j2\pi ynT_1}\\&=\sum_{n=0}^{N_{\rm p}^{\rm c}-1}\overline{\beta}_n(-y)e^{-j2\pi ynT_1}\sum_{i=-\infty}^{\infty}e^{-j2\pi yiN_{\rm p}^{\rm c}T_1}\\&=\sum_{n=0}^{N_{\rm p}^{\rm c}-1}\overline{\beta}_n(-y)e^{-j2\pi ynT_1}\frac{1}{N_{\rm p}^{\rm c}T_1}\sum_{k=-\infty}^{\infty}\delta_{\rm D}\left(y-\frac{k}{N_{\rm p}^{\rm c}T_1}\right)\end{aligned} \tag{86}$$

where the second equality follows from the Poisson sum formula of (83). Substituting (86) into (85) and integrating over variables y and z yields, after simplifications as shown in (87), at the bottom of the page.

Appendix III

Derivation of the Continuous PSD

The continuous PSD of $s(t)$ is given by (88), shown at the bottom of the page. For a real pulse waveform $w(t)$, $|W(f)|=|W(-f)|$, and note that

$$\begin{aligned}\left\langle e^{j2\pi k/(N_{\rm p}^{\rm c}T_1)t}\right\rangle&=\frac{1}{N_{\rm p}^{\rm c}T_1}\int_{-N_{\rm p}^{\rm c}T_1/2}^{N_{\rm p}^{\rm c}T_1/2}e^{j2\pi k/(N_{\rm p}^{\rm c}T_1)t}dt\\&=\begin{cases}1, & k=0\\0, & k\neq 0.\end{cases}\end{aligned} \tag{89}$$

Therefore

$$\begin{aligned}S_s^{\rm c}(f)=&\frac{1}{T_1}\left|W(f)\right|^2\\&\times\sum_{l=-\infty}^{\infty}\left[\frac{1}{N_{\rm p}^{\rm c}}\sum_{n=0}^{N_{\rm p}^{\rm c}-1}K_\beta(n,l,f,f)\right]e^{-j2\pi flT_1}.\end{aligned} \tag{90}$$

$$\begin{aligned}\overline{s}(t)\overline{s}^*(t+\tau)=&\sum_{n=-\infty}^{\infty}\int_y W(y)\mathbb{E}\left\{a_ne^{-j2\pi y(b_nT_2+\epsilon_n)}\right\}e^{-j2\pi yc_nT_3}e^{-j2\pi ynT_1}e^{+j2\pi yt}dy\\&\times\sum_{m=-\infty}^{\infty}\int_z W^*(z)\mathbb{E}\left\{a_m^*e^{+j2\pi z(b_mT_2+\epsilon_m)}\right\}e^{+j2\pi zc_mT_3}e^{+j2\pi zmT_1}e^{-j2\pi zt}e^{-j2\pi z\tau}dz\end{aligned}$$

$$\begin{aligned}\overline{s}(t)\overline{s}^*(t+\tau)=&\frac{1}{N_{\rm p}^{\rm c}T_1}\sum_{k=-\infty}^{\infty}W\left(\frac{k}{N_{\rm p}^{\rm c}T_1}\right)\sum_{n=0}^{N_{\rm p}^{\rm c}-1}\overline{\beta}_n\left(-\frac{k}{N_{\rm p}^{\rm c}T_1}\right)e^{-j2\pi(k/N_{\rm p}^{\rm c})n}\\&\times\frac{1}{N_{\rm p}^{\rm c}T_1}\sum_{l=-\infty}^{\infty}W^*\left(-\frac{l}{N_{\rm p}^{\rm c}T_1}\right)\sum_{m=0}^{N_{\rm p}^{\rm c}-1}\overline{\beta}_m^*\left(\frac{l}{N_{\rm p}^{\rm c}T_1}\right)e^{-j2\pi(l/N_{\rm p}^{\rm c})m}\\&\times\exp\left\{+j2\pi\left(\frac{k+l}{N_{\rm p}^{\rm c}T_1}\right)t\right\}\exp\left\{+j2\pi\left(\frac{l}{N_{\rm p}^{\rm c}T_1}\right)\tau\right\}\end{aligned} \tag{87}$$

$$\begin{aligned}S_s^{\rm c}(f)=&\mathcal{F}_\tau\left\{\langle K_s(t;\tau)\rangle_t\right\}=\sum_{l=-\infty}^{\infty}\int_y\int_z W(y)W^*(z)e^{+j2\pi zlT_1}\\&\times\sum_{k=-\infty}^{\infty}\delta_{\rm D}\left(y-z-\frac{k}{N_{\rm p}^{\rm c}T_1}\right)\frac{1}{N_{\rm p}^{\rm c}T_1}\sum_{n=0}^{N_{\rm p}^{\rm c}-1}K_\beta(n;l,-y,-z)\,e^{-j2\pi(y-z)nT_1}\left\langle e^{+j2\pi(y-z)t}\right\rangle\underbrace{\mathcal{F}\left\{e^{-j2\pi z\tau}\right\}}_{=\delta_{\rm D}(f+z)}dydz\\=&\sum_{l=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}W\Big(-f+\frac{k}{N_{\rm p}^{\rm c}T_1}\Big)W^*(-f)\,e^{-j2\pi flT_1}\frac{1}{N_{\rm p}^{\rm c}T_1}\sum_{n=0}^{N_{\rm p}^{\rm c}-1}K_\beta\Big(n;l,f-\frac{k}{N_{\rm p}^{\rm c}T_1},f\Big)\,e^{-j2\pi(k/N_{\rm p}^{\rm c})n}\left\langle e^{+j2\pi k/(N_{\rm p}^{\rm c}T_1)t}\right\rangle\end{aligned} \tag{88}$$

$$\begin{aligned}S_s^{\mathrm{d}}(f) =& \mathcal{F}_\tau \left\{ \langle \overline{s}(t) \overline{s}^*(t+\tau) \rangle_t \right\} \\ =& \frac{1}{N_{\mathrm{p}}^{\mathrm{c}} T_1} \sum_{k=-\infty}^{\infty} W\left(\frac{k}{N_{\mathrm{p}}^{\mathrm{c}} T_1}\right) \sum_{n=0}^{N_{\mathrm{p}}^{\mathrm{c}}-1} \overline{\beta}_n \left(-\frac{k}{N_{\mathrm{p}}^{\mathrm{c}} T_1}\right) e^{-j2\pi (k/N_{\mathrm{p}}^{\mathrm{c}})n} \\ & \times \frac{1}{N_{\mathrm{p}}{}^{rmc} T_1} \sum_{l=-\infty}^{\infty} W^*\left(-\frac{l}{N_{\mathrm{p}}^{\mathrm{c}} T_1}\right) \sum_{m=0}^{N_{\mathrm{p}}^{\mathrm{c}}-1} \overline{\beta}_m^* \left(\frac{l}{N_{\mathrm{p}}^{\mathrm{c}} T_1}\right) e^{-j2\pi (l/N_{\mathrm{p}}^{\mathrm{c}})m} \left\langle e^{+j2\pi \left((k+l)/(N_{\mathrm{p}}^{\mathrm{c}} T_1)\right) t} \right\rangle \underbrace{\mathcal{F}_\tau \left\{ e^{+j2\pi l/(N_{\mathrm{p}}^{\mathrm{c}} T_1) \tau} \right\}}_{=\delta_{\mathrm{D}}\left(f - l/(N_{\mathrm{p}}^{\mathrm{c}} T_1)\right)}\end{aligned}$$

$$S_s^{\mathrm{d}}(f) = \frac{1}{(N_{\mathrm{p}}^{\mathrm{c}} T_1)^2} \sum_{l=-\infty}^{\infty} \left| W\left(\frac{l}{N_{\mathrm{p}}^{\mathrm{c}} T_1}\right) \right|^2 \left| \sum_{n=0}^{N_{\mathrm{p}}^{\mathrm{c}}-1} \overline{\beta}_n \left(\frac{l}{N_{\mathrm{p}}^{\mathrm{c}} T_1}\right) e^{j2\pi (l/N_{\mathrm{p}}^{\mathrm{c}})n} \right|^2 \delta_{\mathrm{D}}\left(f - \frac{l}{N_{\mathrm{p}}^{\mathrm{c}} T_1}\right) \tag{91}$$

APPENDIX IV

VIII. DERIVATION OF DISCRETE PSD

The discrete PSD of $s(t)$ is given by the first equation shown at the top of the page. Using (89), the above expression reduces to (91), shown at the top of the page.

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